

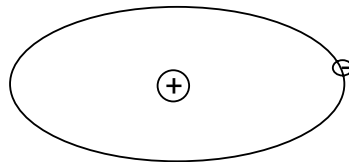
SESSION 1: The Electronic Structure of Atoms

Overview: In this session, we will be looking at:

- **the types of forces** that exist between the proton and electron in the hydrogen atom, **and their importance**.
- the applicability of the Heisenberg Uncertainty Principle to objects of various sizes.

The Hydrogen Atom

(a) Orbits and Classical views of the atom: The Hydrogen (H) atom consists of two particles, a positively charged proton and a negatively charged electron, moving about one another, under the influence of their mutual electrostatic attraction. In this classical view of the hydrogen atom, (with the electron moving around the proton in a fixed orbit), the electron should dissipate (lose) energy in the form of radiation as it moves and therefore follow a spiral path and collapse into the positive nucleus. This does not happen and so classical physics (a branch of Physics) does not explain the behaviour of the electron in the atom.



Electrostatic attraction is similar to gravitational attraction in that the force involved in each case is inversely proportional to the square of the separating distance. (You are asked to calculate and compare these forces in the tutorials!)

(i) *Electrostatic Force of Attraction:*

If two (opposite) charges q_1 and q_2 , in a vacuum, are separated by a distance r , then the electrostatic force (of attraction) between them is given by

$$F_E = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \quad \dots(1)$$

where ϵ_0 ($= 8.854 \times 10^{-12} \text{ C}^2 \text{ J}^{-1} \text{ m}^{-1}$) is the permittivity of a vacuum.

(ii) *Gravitational Force of Attraction:*

If two bodies of mass m_1 and m_2 , are separated by a distance r , then the gravitational force of attraction between them is given by

$$F_G = \frac{Gm_1m_2}{r^2} \quad \dots(2)$$

where G ($= 6.672 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$) is the gravitational constant.

Example 1. Calculate the electrostatic force of attraction between two point charges, +20 C and -60 C separated by 0.20 m in a vacuum.

Solution

Using equation (1)

$$\begin{aligned} F_E &= \frac{q_1q_2}{4\pi\epsilon_0r^2} \\ &= \frac{(+20\text{C})(-60\text{C})}{4(3.142)(8.854 \times 10^{-12} \text{ C}^2 \text{ J}^{-1} \text{ m}^{-1})(0.20 \text{ m})^2} \\ &= 2.69 \times 10^{14} \text{ N} \end{aligned}$$

Example 2. Calculate the gravitational force of attraction between the earth and the moon, making use of the following data:

Mass of earth, $m_{\text{earth}} = 5.97 \times 10^{24} \text{ kg}$

Mass of moon, $m_{\text{moon}} = 7.36 \times 10^{22} \text{ kg}$

separating distance $r = 3.84 \times 10^8 \text{ m}$.

Solution

From equation (2):

$$\begin{aligned} F_G &= \frac{Gm_1m_2}{r^2} \\ &= \frac{(6.672 \times 10^{-11} \text{ N.m}^2 \text{ kg}^{-2})(5.97 \times 10^{27} \text{ kg})(7.36 \times 10^{22} \text{ kg})}{(3.84 \times 10^8 \text{ m})^2} \\ &= 1.99 \times 10^{23} \text{ N} \end{aligned}$$

In the H-atom, the electrostatic force is $\sim 10^{39}$ times the gravitational force! Hence the gravitational force is considered negligible and insignificant.

According to Classical Mechanics, used to explain the motion of large bodies, the proton-electron system should behave similarly to that of the Earth – Moon system i.e. they should both move about a common centre of gravity in circles or ellipses.

The mass of the proton is approximately 2000 times (1836 times to be exact) the mass of the electron. In this case, the centre of gravity of the proton-electron system is so close to (actually inside) the proton that the proton can be assumed to be in a fixed position and the electron orbiting it. If the position q , and velocity v , of the electron are known at some time, then we should be able to calculate the orbit in which it is moving and predict its future motion.

This is the point where we run into serious problems, as it is impossible to measure both the position and velocity of the electron, simultaneously, with any reasonable accuracy. If one is determined accurately then the accuracy with which the other is measured is sacrificed.

This leads us to the **Heisenberg Uncertainty Principle (HUP)**, which explains this observation:

The Heisenberg Uncertainty Principle (HUP)

If q and v are the position and velocity of an electron respectively and δq and δv are their respective uncertainties then, by the HUP,

$$\delta q \cdot \delta v \geq \frac{\hbar}{2m_e} \quad \dots(3)$$

$$\left(\hbar = \frac{h}{2\pi} \right)$$

where h , Planck's constant = 6.626×10^{-34} J s, and
 m_e , mass of the electron = 9.109×10^{-31} kg.

If one looks closely at the inequality (3), it can be seen that the right hand side is a constant (for a given mass), which is to say that, the product of the uncertainties cannot be less than the given value of $h/4\pi m_e$. If δq is very small (i.e. q is known with high accuracy) then δv has to increase in value in order to maintain the inequality, and *vice versa*.

The inequality can be rearranged and written as

$$\delta q \cdot \delta p \geq h/4\pi \quad \dots(4)$$

where $p (= mv)$ is the linear momentum, and $\delta p = m \delta v$.

We will now draw on some experimental evidence, which will help to shed some light on the matter of determining both the electron's position and momentum simultaneously with great accuracy. To locate the electron, one has to use photons of a certain minimum energy.

Example 3. Determine the uncertainty in the path of flight of a 10.0 g-bullet if its velocity is $500 \pm 1 \text{ m s}^{-1}$.

(N.B. In doing calculations, if all parameters are expressed in SI units then the answer will come out with SI units).

Solution

From equation (4),

$$\delta q \cdot \delta p \geq h/4\pi$$

but $\delta p = \delta(mv) = m\delta v$ (assuming that the uncertainty in the momentum is as a result of the uncertainty in the velocity, and also assuming negligible uncertainty in the mass).

$$\text{and, } \delta q \cdot \delta v \geq h/4\pi m$$

$$\text{and } \delta q \geq h/4\pi m \delta v.$$

$$\begin{aligned} \delta q &\geq \frac{6.626 \times 10^{-34} \text{ J s}}{4 \times 3.142 \times 0.0100 \text{ kg} \times 1 \text{ m s}^{-1}} \\ &= 5.27 \times 10^{-29} \text{ m} \end{aligned}$$

This result tells us that there is very small deviation of the flight path (predicted position) of the bullet, which allows for high accuracy in prediction of the bullet's position. Hence, a marksman is able shoot a 'bullseye'! This small uncertainty is associated with objects much larger than electrons, atoms and molecules.

Remember that all electromagnetic radiation consists of photons and the energy of a photon is given by

$$E = h\nu \quad \dots(5)$$

where ν is the frequency (in s^{-1}) of the radiation.

Locating the electron with photons can be compared to locating a mosquito in the night with the light from a flashlight. Both cases are using photons to locate an object, but in the case of the mosquito the photons have a comparatively much smaller mass (much smaller momentum). This means that the momentum from the light photons will not be enough to significantly disturb the flight path of the mosquito, and one can see EXACTLY where the mosquito is located at some specific point in time. Let us now turn our attention to the case of the electron, which is a much, much smaller mass with a much, much smaller momentum. To locate the electron with high accuracy we need to use photons of high energy. High energy, here, means photons of higher velocity and hence greater momentum, which will offer greater disturbances, upon impact, to the path of the electron. If we need to be more accurate in locating the electron, we will have to use photons of higher energy, and then there will be even more disturbances in the momentum of the electron. To cause fewer disturbances in the momentum then we will have to use photons of lower energy, and this will cause us to locate (see) the electron less accurately. Locating the electron with pinpoint accuracy (i.e. $\delta q = 0$) would require $\delta v = \infty$, which would mean that we know nothing at all about its velocity.

This difficulty or uncertainty exists for all systems. For the earth – moon system where the mass of the moon ($m_{\text{moon}} = 7.4 \times 10^{22}$ kg), $\delta q \cdot \delta v$ is negligible in the case of the moon, but is significant in the case of the proton-electron system where the mass of the electron is 9.1×10^{-31} kg. This implies that the orbit (or path) of the electron cannot be calculated (with a reasonable degree of accuracy).

ALL IS NOT LOST!

At this point it would seem useless to investigate the matter any further, but as scientists we seek out another observable which we can determine with greater accuracy and try and get as much information from (or about) it as possible. There are other properties of the H atom, which can be measured accurately, and do not depend on the simultaneous position and velocity of the electron. Such a useful physical property is *energy*.

The ***Law of Conservation of Energy*** tells us that the total energy of an *isolated* H atom must have some fixed value, regardless of the positions and velocities of the proton and electron.

$$\text{Total Energy} = \text{Kinetic Energy}_{\text{total}} + \text{Potential Energy}_{\text{total}} \quad (6)$$

(due to motion) (due to attraction)

The HUP restricts the determination of the total energy of the proton and of the electron separately *but we can determine the sum*. So, the logical thing to do here is to make some observations on the atom and its components while maintaining the total energy at a constant value.