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DEPARTMENT OF CHEMISTRY

C10J ATOMIC STRUCTURE (Tutorial #1 : Solutions)

1. The gravitational force of attraction between two (2) bodies of masses m_1 and m_2 separated by a distance, r , is given by the equation
- $$F = Gm_1m_2/r^2$$
- $$\therefore F = Gm_p m_e / r^2$$
- $$\therefore F = (6.672 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(1.673 \times 10^{-27} \text{ kg})(9.109 \times 10^{-31} \text{ kg}) / (5.292 \times 10^{-11} \text{ m})^2$$
- $$= \underline{\underline{3.631 \times 10^{-47} \text{ N}}}$$

The electrostatic force of attraction between two particles with charges q_1 and q_2 separated by a distance, r , is given by the equation

$$F = q_1q_2/4\pi\epsilon_0r^2$$
$$\therefore F = q_1q_2/4\pi\epsilon_0r^2$$
$$\therefore F = q_eq_p/4\pi\epsilon_0r^2$$
$$= (-1.602 \times 10^{-19} \text{ C})(1.602 \times 10^{-19} \text{ C}) / (1.113 \times 10^{-10} \text{ C}^2 \text{ m}^{-1} \text{ J}^{-1})(5.292 \times 10^{-11} \text{ m})^2$$
$$= \underline{\underline{-8.24 \times 10^{-8} \text{ N}}}$$

The $-ve$ sign signifies an attractive force. The gravitational force of attraction is negligible when compared to the electrostatic force. The electrostatic force of attraction is much larger and will be the force that dictates the motion of the electron.

2. When two masses are attracted to each other and rotational motion is present, then they will both rotate about the common centre of gravity. If the masses are equal, then the centre of gravity is mid-way between the two. If one mass is larger then the centre of gravity is closer to it, the distance being in an inverse ratio to the mass. For example, if two masses m_1 and m_2 are at distances r_1 and r_2 , respectively, from their common centre of gravity then

$$\mathbf{m_1r_1 = m_2r_2 \text{ and } m_1/m_2 = r_2/r_1}$$

This means that for the hydrogen atom system, the proton being much more massive than the electron will mean that the centre of gravity is essentially at the centre of the proton and the proton will therefore rotate about its own centre (like a spinning top).

3. The principle essentially states that for very small particles (electrons, protons, neutrons, single atoms, etc) it is impossible to measure both the velocity and the position with any appreciable accuracy at the same time. It is possible to measure one accurately but the error or uncertainty in the other will, necessarily, be large.

If the uncertainty in the velocity, v , is Δv and the uncertainty in the position, q , is Δq for a mass m then

$$\Delta v \cdot \Delta q \geq h/4\pi m \quad (1)$$

or

$$\Delta p \cdot \Delta q \geq h/4\pi, \quad (2)$$

since $m\Delta v = \Delta p$ (linear momentum, $p = mv$)

Take equation (1) for example, the right-hand side of the inequality has some fixed value for some mass m . If the velocity is measured accurately, which means that Δv is very small, then the uncertainty in the position, Δq , would have to increase in value so as to maintain the inequality, hence the accuracy of the position suffers.

4. Again equation (1) above can shed some light on the matter.

$$\Delta v \cdot \Delta q \geq h/4\pi m$$

The mass of the moon is $\sim 10^{28}$ g and the mass of the electron is $\sim 10^{-28}$ g. If these masses are substituted in the inequality, it can be seen that in the case of the moon the right-hand side (RHS) will be much smaller than in the case of the electron. The extremely small RHS in the case of the moon means that both Δv and Δq must be small, in other words, both v and q can be measured accurately at the same time. Hence, the HUP is less important for large objects.

- 5 (a) $\Delta p \cdot \Delta q \geq h/4\pi$

$$\Delta q = 10^{-12} \text{ m}$$

Rearrange the inequality to give

$$\Delta p \geq h/4\pi(\Delta q)$$

$$\geq (6.626 \times 10^{-34} \text{ Js}) / (4 \times 3.142 \times 10^{-12} \text{ m})$$

$$\geq \underline{\underline{5.28 \times 10^{-23} \text{ kg m s}^{-1}}}$$

- (b) $p = mv$

$$= (9.109 \times 10^{-31} \text{ kg})(2.998 \times 10^8 \text{ m s}^{-1})/3$$

$$= \underline{\underline{9.11 \times 10^{-23} \text{ kg m s}^{-1}}}$$

When the value of the momentum from (b) is compared with the value of the uncertainty in the momentum from (a) this uncertainty is quite large (i.e. there is a large error in the value of the momentum). This just shows that if the position of the electron is known with reasonable accuracy (i.e. $\pm 10^{-12}$ m) then the accuracy with which the momentum can be known suffers tremendously.

The diameter of the H-atom is $\sim 10^{-10}$ m (1 Å). What the above result is saying to us is that we are not even allowed to pin-point the location of the electron to within one-hundredth of the diameter of the atom.

6. Covered in notes.
7. Covered in notes.