M10B Stream(a) **PROBLEM PAPER 7** **Due: 25/03/2008**

1) a) Prove by induction that, for any natural number n,

$$\sum_{k=1}^{n} \frac{k}{3^k} = \frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \frac{4}{81} + \cdots + \frac{n}{3^n} = \frac{3}{4} - \frac{2n+3}{4 \cdot 3^n}$$

b) Hence determine whether $\sum_{k=1}^{\infty} \frac{k}{3^k}$ converges and, if so, find its sum.

2) Let (s_n) be the sequence such that $s_n = \frac{n+1}{3n+5}$ for all n.

a) Find a series $\sum_{k=1}^{\infty} a_k$ whose sequence of partial sums is (s_n).

b) Determine whether the series of part (a) above converges. [Justify your answer.]

3) For each positive series below, use a valid test to determine whether it is summable.

a) $\sum_{k=1}^{\infty} \frac{7 \cdot 2^k}{3 \cdot 5^k}$

b) $\sum_{k=1}^{\infty} \frac{7 \cdot 2^k}{3 + 5^k}$

c) $\sum_{k=1}^{\infty} \frac{3k}{4k+5}$

d) $\sum_{k=1}^{\infty} \frac{6}{4k-1}$

e) $\sum_{k=1}^{\infty} \frac{1}{4^{1/k}}$

f) $\sum_{k=1}^{\infty} \frac{3^{1/k}}{4^k}$

4) In each part below, either give an example of a series $\sum_{k=1}^{\infty} a_k$ with both of the stated properties or explain briefly why no such series exists.

a) $\lim_{k\to\infty} a_k = 2$; $\sum_{k=1}^{\infty} a_k = 2$.

b) $\lim_{k\to\infty} a_k = 0$; $\sum_{k=1}^{\infty} a_k = 4$.

c) $\lim_{k\to\infty} a_k = 0$; $\sum_{k=1}^{\infty} a_k$ diverges to ∞.