M10B Stream(a) **SOLUTIONS 1** **01/25/2008**

1(a) To solve $|x_n - 0| < \dfrac{1}{1,000}$ we set

$$\left|\frac{5}{\sqrt{n^2+4}} - 0\right| < \frac{1}{1,000}; \qquad \frac{5}{\sqrt{n^2+4}} < \frac{1}{1,000}; \qquad 5,000 < \sqrt{n^2+4}$$

Squaring both sides,

$$25,000,000 < n^2 + 4; \qquad 24,999,996 < n^2; \qquad 4,999.9996 < n.$$

Thus $m = 5,000$ is the smallest natural number for which $|x_m - 0| < 1/1,000$.

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1(b) Let $\epsilon > 0$, choose N such that $N > \sqrt{\dfrac{25}{\epsilon^2} - 4}$, and let n be given. If $n > N$ then $n > \sqrt{\dfrac{25}{\epsilon^2} - 4}$, hence

$$\sqrt{\frac{25}{\epsilon^2} - 4} < n; \qquad \frac{25}{\epsilon^2} - 4 < n^2; \qquad \frac{25}{\epsilon^2} < n^2 + 4;$$

$$\frac{5}{\epsilon} < \sqrt{n^2+4}; \qquad \frac{5}{\sqrt{n^2+4}} < \epsilon; \qquad \left|\frac{5}{\sqrt{n^2+4}} - 0\right| < \epsilon.$$

In summary, if $n > N$ then $\left|\dfrac{5}{\sqrt{n^2+4}} - 0\right| < \epsilon$. Thus $\left(\dfrac{5}{\sqrt{n^2+4}}\right) \to 0$.

2(a) To solve $|x_n - 3| < \dfrac{1}{2,500}$ we set

$$\left|\frac{3n+4}{n+6} - 3\right| < \frac{1}{2,500}; \qquad \left|\frac{3n+4}{n+6} - \frac{3n+18}{n+6}\right| < \frac{1}{2,500};$$

$$\left|\frac{-14}{n+6}\right| < \frac{1}{2,500}; \qquad \frac{14}{n+6} < \frac{1}{2,500}; \qquad 35,000 < n + 6; \qquad 34,994 < n.$$

Thus $m = 34,995$ is the smallest natural number for which $|x_m - 3| < 1/2,500$.

2(b) Let $\epsilon > 0$, choose N such that $N > \dfrac{14}{\epsilon} - 6$, and let n be given. If $n > N$ then $n > \dfrac{14}{\epsilon} - 6$, hence

$$\frac{14}{\epsilon} - 6 < n; \qquad \frac{14}{\epsilon} < n + 6; \qquad \frac{14}{n+6} < \epsilon; \qquad \left|\frac{-14}{n+6}\right| < \epsilon;$$

$$\left|\frac{3n+4-3n-18}{n+6}\right| < \epsilon; \qquad \left|\frac{3n+4}{n+6} - \frac{3n+18}{n+6}\right| < \epsilon; \qquad \left|\frac{3n+4}{n+6} - 3\right| < \epsilon.$$

In summary, if $n > N$ then $\left|\dfrac{3n+4}{n+6} - 3\right| < \epsilon$. Thus $\left(\dfrac{3n+4}{n+6}\right) \to 3$.

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(3) Let $\epsilon > 0$, choose any N you please (for example, choose $N = 1$), and let n be given. If $n > N$ then

$$|x_n - L| = |c - c| = |0| = 0 < \epsilon.$$

In summary, if $n > N$ then $|c - c| < \epsilon$. Thus $(c) \to c$.

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(4) Let $\epsilon > 0$, choose N so large that $N > \dfrac{1}{\sqrt{\epsilon}}$, and let n be given. If $n > N$ then $n > \dfrac{1}{\sqrt{\epsilon}}$, hence

$$\frac{1}{\sqrt{\epsilon}} < n; \qquad \frac{1}{\epsilon} < n^2; \qquad \frac{1}{n^2} < \epsilon; \qquad \left|\frac{1}{n^2}\right| < \epsilon.$$

Since $|\sin n| \le 1$ for all n, it follows that $\left|\dfrac{\sin n}{n^2} - 0\right| = \left|\dfrac{\sin n}{n^2}\right| \le \left|\dfrac{1}{n^2}\right| < \epsilon$. In summary, if $n > N$ then $\left|\dfrac{\sin n}{n^2} - 0\right| < \epsilon$. Thus $\left(\dfrac{\sin n}{n^2}\right) \to 0$.

5(a) First,

$$\frac{n^2+2}{3n^2+4n+1} = \frac{n^2+2}{3n^2+4n+1} \times \frac{1/n^2}{1/n^2} = \frac{1+2/n^2}{3+4/n+1/n^2}$$

Then, referring to the numerator, $(1) \to 1$ and $(2/n^2) \to 0$, hence $(1 + 2/n^2) \to 1 + 0 = 1$. Also, referring to the denominator, $(3) \to 3$, $(4/n) \to 0$, and $(1/n^2) \to 0$. It follows that $(3 + 4/n + 1/n^2) \to 3 + 0 + 0 = 3$. Finally, since $3 + 4/n + 1/n^2 \neq 0$ for all n and $3 \neq 0$,

$$\left(\frac{1+2/n^2}{3+4/n+1/n^2}\right) \to \frac{1}{3}.$$

That is, $\left(\dfrac{n^2+2}{3n^2+4n+1}\right) \to \dfrac{1}{3}$.

. . . .

5(b) First,

$$\frac{n^2+2}{3n^3+4n+1} = \frac{n^2+2}{3n^3+4n+1} \times \frac{1/n^3}{1/n^3} = \frac{1/n+2/n^3}{3+4/n^2+1/n^3}$$

Then, referring to the numerator, $(1/n) \to 0$ and $(2/n^3) \to 0$, hence $(1/n+2/n^3) \to 0+0=0$. Also, referring to the denominator, $(3) \to 3$, $(4/n^2) \to 0$, and $(1/n^3) \to 0$. It follows that $(3+4/n^2+1/n^3) \to 3+0+0=3$. Finally, since $3+4/n^2+1/n^3 \neq 0$ for all n and $3 \neq 0$,

$$\left(\frac{1/n+2/n^3}{3+4/n^2+1/n^3}\right) \to \frac{0}{3} = 0.$$

That is, $\left(\dfrac{n^2+2}{3n^3+4n+1}\right) \to 0$.