

# PUERTO RICAN MATHEMATICAL OLYMPIAD

## ROUND II

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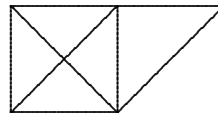
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## Second Round Questions

### Level I (Grades 4–6)

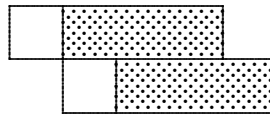
- 1) Rosita has a large box with 4 medium-sized boxes inside; in each of the medium-sized boxes there are two small boxes, and in each small box there are three tiny boxes. How many boxes does Rosita have?
- 2) Which number is missing in the following progression: 1, 3, 7, 15, \_\_\_\_, 63, 127?
- 3) How many triangles are in the following figure?



- 4) Antonio, Beatrice, Carlos, and Diane were seated in a line with four chairs numbered from 1 to 4. Emilio came by and said,
  - Beatrice is beside Carlos;
  - Antonio is between Beatrice and Carlos.

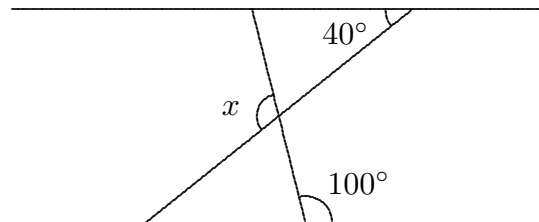
If both statements are false and Beatrice is in Chair 3, who sits in Chair 2?

- 5) An animal trainer has three tigers and two lions. He would like to line them up in such a way that no two lions and no two tigers are together. If each lion and tiger is distinct from the others, in how many ways can he line up his animals?
- 6) Which number multiplied by 3 corresponds to three-fourths of 120?
- 7) Each white square below has a side length of 6 cm, and the perimeter of each shaded rectangle is double that of a white square. What is the perimeter of the figure below?



- 8) On a certain planet, the number of days in a week are the same as the number of weeks in a month, and this is the same as the number of months in a year. If there are 1000 days in a year, how many days are in one week?
- 9) A cake is cut and  $\frac{1}{3}$  of it is removed. Then it is cut again and  $\frac{1}{3}$  of the remaining portion is removed; then it is cut one more time and  $\frac{1}{3}$  of the remaining part is removed. What proportion of the original cake remains after the third cut?

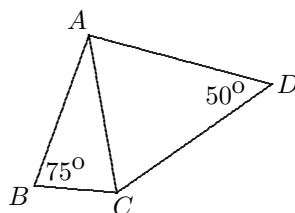
- 10) On Wednesday, everyone at a canteen receives a 10% discount but then has to pay 10% tax on the balance. What is the final price as a percentage of the original price?
- 11) If you multiply all the odd numbers between 1 and 2008, what is the last digit of the product?
- 12) A certain movie theatre has 26 rows with 24 seats in each row. If they are numbered from left to right, starting from the first row and proceeding toward the rear, in which row is seat number 375?
- 13) Maria had the number 4921508 and asked Juan to erase some of the digits in such a way that the resulting number had three digits and was as small as possible. What number did Juan obtain?
- 14) What is the last digit of  $2^{2008} - 2$ ?
- 15) In the diagram below, the horizontal lines are parallel. What is the measure of  $\angle x$ ?



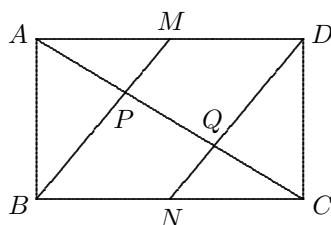
## Second Round Questions

### Level II (Grades 7–12)

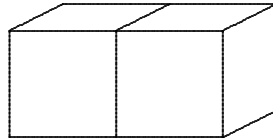
- 1) Maria gets channels 2 through 42 on her television. When scrolling through the channels they cycle so that after channel 42 comes channel 2 again. If she is watching Channel 15 and advances through 518 channels, which one will she be watching?
- 2) Last Sunday, Marta received several visitors. When Pedro arrived, Raul was already there. Jesus and Rita arrived together. Luisa opened the door for Arthur and Arthur opened the door for Rita. Raul arrived after Rita. Who was the last one to arrive?
- 3) Some pigeons and piglets are in a cage. If they have a total of 14 eyes and 22 feet, how many pigeons and how many piglets are in the cage?
- 4) On a piece of paper there is a three-digit number ending with a 2. If that 2 is removed and placed at the beginning of the number, the result is a three-digit number which is 36 less than the original. What is the sum of the digits of the original number?
- 5) In the figure below,  $AD = CD$ ,  $AB = AC$ , angle  $ABC$  is  $75^\circ$ , and angle  $ADC$  is  $50^\circ$ . What is the measure of angle  $BAD$ ?



- 6) Julio wrote a secret number on a piece of paper and observed that his number has four digits which sum to 9 and none of them are 0; also, the number is a multiple of 5 and it is more than 1995. What is the third digit (from right to left) of his secret number?
- 7) In the rectangle below,  $M$  and  $N$  are the midpoints of  $AD$  and  $BC$ , respectively, and  $AC$  intersects  $MB$  and  $DN$  at  $P$  and  $Q$ , respectively. If  $AD$  is 5 cm and  $AB$  is 3 cm, what is the area of the quadrilateral  $MPQD$  in square centimetres?



- 8) A large bag contains marbles of 20 different colours. Some are removed at random. What is the least number of marbles that must be removed in order to guarantee that you will have drawn at least 100 marbles of the same colour?
- 9) Juan eliminated one number from a list of 10 consecutive numbers. If the sum of the remaining numbers was 2008, which number was eliminated?
- 10) If you write down all of the numbers from 1 to 1000, how many times will you write the digit 5?
- 11) A certain cube with sides of length 4 inches is made up of cubes with sides of length 1 inch. How many 1-inch cubes are in contact with exactly 4 other such cubes? [We say that two cubes are in contact when they share a common face, as shown below.]



- 12) A number is called “stuttering” if all of its digits are 1. How many whole numbers less than 100,000 are there which can be multiplied by 33 to give a stuttering number?
- 13) In how many ways can you choose one white and one black square on a chess board so that they are neither in the same row nor in the same column?
- 14) In the equation  $1 + (n^2 + n)(n^2 + 5n + 6) = 181^2$ , where  $n$  is a whole number, what is the value of  $n(n + 3)$ ?
- 15) Suppose two circles have the same centre and the area between them (which is inside the larger circle and outside the smaller circle) is  $\frac{25\pi}{2}$  cm<sup>2</sup>. What is the length of a chord across the larger circle if it is tangent to the smaller circle?