

PUERTO RICAN MATHEMATICAL OLYMPIAD

GRADES 4, 5, AND 6

ROUNDS I AND II

2007/2008

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Preface

Each year, many countries around the world participate in the International Mathematical Olympiad. This is a mathematics problem-solving competition in which high school students tackle significant problems over a two-day period. In 2010, the Olympiad was held in Kazakhstan and 97 countries participated.

Each year, many countries also participate in various Regional Olympiads. These Olympiads encourage students to enter their national programmes and prepares them for the highest level of competition. In 2010, a team of Jamaican high school students participated in a Regional Mathematical Olympiad for the first time ever. This was held in Puerto Rico and featured 14 countries from Central America and the Caribbean. These countries included 11 established teams and 3 competing for the first time. The Jamaican team finished 11th, which was below most established teams but best among the new teams.

In order to build for the future, the Department of Mathematics at the UWI, Mona, is launching a Junior Mathematical Olympiad. This is a national competition for students in Grades 4, 5, and 6 and will be held in three rounds. This is sponsored by the University of the West Indies, Mona, and the Insurance Association of Jamaica. The name of the programme will commemorate Vivian Rochester. He was a well regarded executive in the insurance industry and helped organize the earliest mathematics competitions in Jamaica.

In order to assist with the preparations, the Alliance for the Improvement of Mathematics Learning (AFAMaC) has kindly provided Jamaica with many materials. This alliance is based in Puerto Rico and is a joint effort between the University of Puerto Rico, Mayaguez, and the Puerto Rico Department of Education. The questions here were used in the first two stages of their 2007/2008 Olympiad. The solutions will be posted on the website of the Department of Mathematics.

On behalf of the Department of Mathematics at the UWI, Mona, I thank the AFAMaC for assisting us in many ways as we prepare for this year's events. In particular, I thank them for their kind permission to use their past questions and circulate them freely in Jamaica. I also wish all students good luck and much success in their mathematical studies this year.

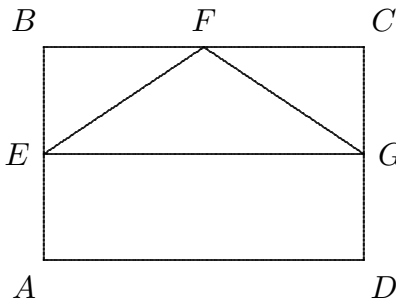
Raymond McEachin

January 17, 2011

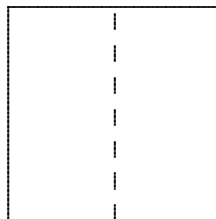
- 10) In the sum below, different letters stand for different digits, AH is a two-digit number, and HEE is a three-digit number. Find $H + E$.

$$\begin{array}{r} AH \\ + A \\ \hline HEE \end{array}$$

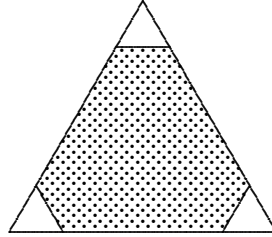
- 11) Suppose we want to paint a cube in such a way that no two adjacent sides have the same colour? What is the least number of colours we would need?
- 12) In the diagram below, the rectangle $ABCD$ has area 36 square units. The points E , F , and G are the midpoints of the sides on which they lie. What is the area of the triangle EFG ?



- 13) If a cube 5 cm on each side is formed of cubes 1 cm on each side, how many of the smaller cubes cannot be seen from the outside?
- 14) Luis wrote down all of the numbers from 1 to 100. What is the total number of digits he wrote down?
- 15) If the base of a triangle is increased by 10% and its altitude is decreased by 10%, by how much is its area changed? [You may give your answer as a percent.]
- 16) One light bulb flashes every 2 minutes and another flashes every $3\frac{1}{2}$ minutes. If both flash at midnight, when is the first time after 1 am that they flash together again?
- 17) A square sheet of paper is folded down the middle to form two rectangles as shown below. If each rectangle has perimeter 18 cm, what is the perimeter of the original square?



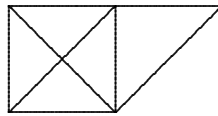
- 18) At the Balboa Theatre, the price of admission is \$7. They have four shows each day. For the first and second shows, admission is half price. One Thursday, the fourth show had twice as many patrons as the third, and the third had twice as many as the first and second shows combined. If they received \$1183 that day, how many patrons were at the fourth show?
- 19) From an equilateral triangle with perimeter 75 cm, an equilateral triangle with sides 5 cm is removed from each corner. What is the perimeter of the resulting region?



- 20) Susana works in a building with 5 offices. In that building, there are three plants: a cactus, an azalea, and an orchid. Each day, Susana changes the plants in the offices. In how many ways can she arrange the plants so that not all three of them are placed in the same office?

Second Round Questions
Grades 4, 5, and 6

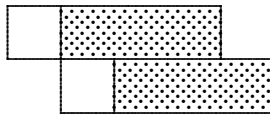
- 1) Rosita has a large box with 4 medium-sized boxes inside; in each of the medium-sized boxes there are two small boxes, and in each small box there are three tiny boxes. How many boxes does Rosita have?
- 2) Which number is missing in the following progression: 1, 3, 7, 15, ____, 63, 127?
- 3) How many triangles may be found in the following figure?



- 4) Antonio, Beatrice, Carlos, and Diane were seated in a line with four chairs numbered from 1 to 4. Emilio came by and said,
 - Beatrice is beside Carlos;
 - Antonio is between Beatrice and Carlos.

If both statements are false and Beatrice is in Chair 3, who sits in Chair 2?

- 5) An animal trainer has three tigers and two lions. He would like to line them up in such a way that no two lions and no two tigers are together. If each lion and tiger is distinct from the others, in how many ways can he line up his animals?
- 6) Which number multiplied by 3 corresponds to three-fourths of 120?
- 7) Each white square below has a side length of 6 cm, and the perimeter of each shaded rectangle is double that of a white square. What is the perimeter of the figure below?



- 8) On a certain planet, the number of days in a week are the same as the number of weeks in a month, and this is the same as the number of months in a year. If there are 1000 days in a year, how many days are in one week?
- 9) A cake is cut and $\frac{1}{3}$ of it is removed. Then it is cut again and $\frac{1}{3}$ of the remaining portion is removed; then it is cut one more time and $\frac{1}{3}$ of the remaining part is removed. What proportion of the original cake remains after the third cut?

- 10) On Wednesday, everyone at a canteen receives a 10% discount but then has to pay 10% tax on the balance. What is the final price as a percentage of the original price?
- 11) If you multiply all the odd numbers between 1 and 2008, what is the last digit of the product?
- 12) A certain movie theatre has 26 rows with 24 seats in each row. If they are numbered from left to right, starting from the first row and proceeding toward the rear, in which row is seat number 375?
- 13) Maria had the number 4921508 and asked Juan to erase some of the digits in such a way that the resulting number had three digits and was as small as possible. What number did Juan obtain?
- 14) What is the last digit of $2^{2008} - 2$?
- 15) In the diagram below, the horizontal lines are parallel. What is the measure of $\angle x$?

