First Round Solutions

Grades 4, 5, and 6

1) There are four basic rectangles not made up of smaller ones. There are three more rectangles made up of two smaller ones each, two rectangles made up of three smaller ones each, and the outer rectangle (which is in fact a square) made up of the four smaller ones. Thus there are $4 + 3 + 2 + 1 = 10$ rectangles in all.

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2) The first, third, fifth, etc., terms are 1, 2, 3, etc. The second, fourth, sixth, etc., terms are 4, 8, 12, etc. Following these progressions separately, the next two terms are 20 and 6.

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3) Reading from right to left, the there are two choices for the last digit, two for the middle digit, and one for the first digit (which must be a 5). Then there are $1 \times 2 \times 2 = 4$ such numbers in total, which are 500, 505, 550, and 555.

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4) If half a pound is removed from each box then a total of 10 lbs is removed from the 20 boxes. The remaining papayas will weigh 1590 lbs.

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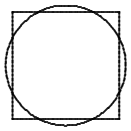
5) In the diagram below, the square intersects the circle in 8 points. This is the maximum because no side of a square can cut a circle in more than two points. Since squares have four sides, no square can intersect a circle in more than eight points.



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6) If there are ten trees then there are nine spaces between them. Since each space is 15 m, the distance between the first and last trees is 135 m.

7) Suppose Maria gives a gift to Ana. Then Ana cannot give one to Maria or else Rosa must give a gift to herself. So Ana must give a gift to Rosa and Rosa must give one to Maria. If Maria doesn't give a gift to Ana then she must give one to Rosa. In this case, Rosa will give one to Ana and Ana will give one to Maria. Thus there are two ways of exchanging gifts at the party.

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8) If there are n dogs and n chickens then there are $4n$ legs on the dogs and $2n$ legs on the chickens. The total number of legs is $6n$. The only possible number of legs is 24 because that is the only 6-multiple among the choices offered.

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9) After n years, Mario was $27 + n$ and Pedro was $3 + n$. Mario would have been three times as old as Pedro when $27 + n = 3(3 + n)$. In this case,

$$27 + n = 9 + 3n; \qquad 18 = 2n; \qquad 9 = n$$

Thus after 9 years Mario was three times as old as Pedro, and this happened when Mario was 36 and Pedro was 12.

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10) Note that AH can be no larger than 99 and A can be no larger than 9. Then HEE must be a number between 100 and 108. In particular, $H = 1$. Also, $A = 9$ because if it were less than 9 then the sum of AH and A would be less than or equal to 89. Then $A = 9$, $AH = 91$, and $HEE = 100$. Thus $H + E = 1$.

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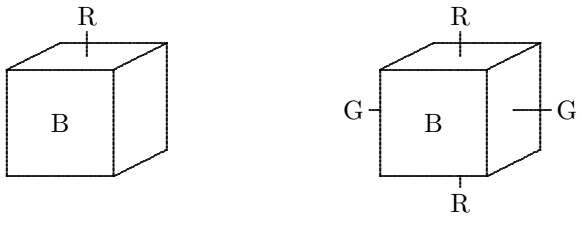
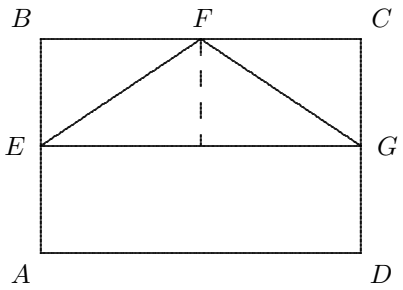
11) Suppose one side, say the top, is red and an adjacent side, say the front, is blue. A side adjacent to both of them must be another colour such as green. However, with these three colours the rest of the cube can be coloured as shown. (The back of the cube would be blue.) The least number of colours needed is 3.



12) The area of rectangle $BCGE$ is half the area of $ABCD$ and the area of triangle EFG is half the area of $BCGE$. Then the area of the triangle EFG is 9 square units.



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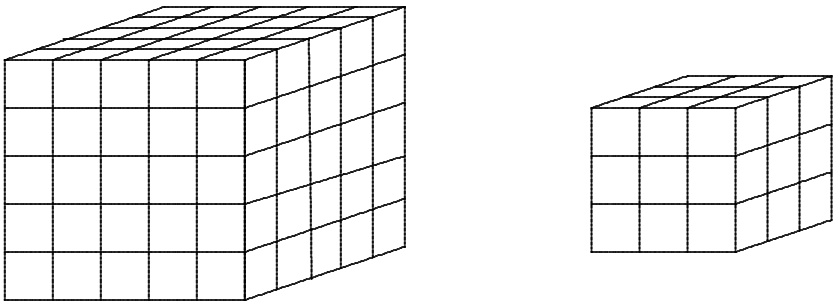
13) If you remove all the small cubes which can be seen from the outside, there would remain a $3 \times 3 \times 3$ cube which could not have been seen from the outside. Thus there are 27 smaller cubes which cannot be seen from the outside.



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14) Luis wrote down 1 digit for each number from 1 to 9, 2 digits for each number from 10 to 99, and three digits for the number 100. Thus he wrote down $9(1) + 90(2) + 1(3) = 192$ digits in total.

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15) Let b and h be the original base and height, respectively, and B and H the new ones. Then $B = b + 0.1b = 1.1b$ and $H = h - 0.1h = 0.9h$. The new area is $\frac{1}{2}BH = \frac{1}{2}(1.1b)(0.9h) = \frac{0.99}{2}bh$. Since the original area is $\frac{1}{2}bh$, the new area is 0.99 or 99% of the original one.

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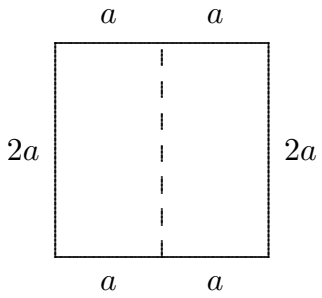
16) The first bulb will flash after 2, 4, 6, 8, 10, 12, 14, ..., minutes. The first time on this list that is a multiple of $3\frac{1}{2}$ minutes is 14 minutes. It follows that the bulbs flash together every 14 minutes. This means that they flash together at 12:14 am, 12:28 am, 12:42 am, 12:56 am, and 1:10 am. The first time after 1 am that they flash together is 1:10 am.

17) Referring to the diagram below, $a + 2a + a + 2a = 18$. Thus $6a = 18$ and hence $a = 3$. It follows that the perimeter of the original square is 24



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18) Let A, B, C, and D be the number of patrons at the four shows that day. Then $C = 2(A + B)$ and $D = 2C = 4(A + B)$. The money the theatre received was

$$3.50A + 3.50B + 7C + 7D = 3.50(A + B) + 14(A + B) + 28(A + B) = 45.50(A + B)$$

Thus $45.5(A + B) = 1183$ and hence $A + B = 26$. There were $4(26) = 104$ patrons at the fourth show.

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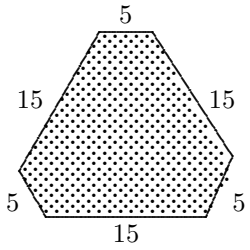
19) Each side of the original triangle is 25 cm long. After removing the smaller triangles each longer side has length 15 cm. Since each shorter side has length 5 cm, the perimeter of the resulting region is 60 cm.



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20) There are two possibilities to consider: three offices contain one plant each or one office contains two plants while another has one. In the first case, one plant may be placed in any of 5 offices, another placed in any of the 4 remaining offices, and the third in any of the last three offices. Thus there are $5 \times 4 \times 3 = 60$ ways of placing the plants this way. In the second case, there are three ways of pairing two plants to be placed in the same office: the cactus and the azalea, the cactus and the orchid, and the azalea and the orchid. Then the pair may be placed in any of 5 offices and the single plant placed in any of the remaining 4 offices. Then there are $3 \times 5 \times 4 = 60$ ways of placing the plants in this case. Combining the two cases, there are $60 + 60 = 120$ ways all together of placing the plants.

Second Round Solutions

Grades 4, 5, and 6

1) Rosita has 1 large box, 4 medium-sized boxes, 8 small boxes, and 24 tiny boxes. The total number of boxes is $1 + 4 + 8 + 24 = 37$.

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2) After the first term, we may add 2, 4, and 8, respectively, to generate the next three terms. If we then add 16, 32, and 64 we would obtain 31, 63, and 127. Since these last two numbers are correct, the missing number is evidently 31.

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3) There are five basic triangles which are not made up of smaller ones. There are four more triangles made up of two smaller ones, and one more triangle made up of three smaller ones. There are a total of 10 triangles in the diagram.

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4) We know that Beatrice is in Chair 3. Also, since Beatrice and Carlos are not beside each other, Carlos is in Chair 1. Since Antonio is not between Beatrice and Carlos, he is in Chair 4. This means that Diane in Chair 2.

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5) If T denotes a tiger and L denotes a lion, the animals must be lined up in the order T, L, T, L, T. There are 3 choices for the first position, 2 for the second, 2 for the third, 1 for the fourth, and 1 for the fifth. The number of possible arrangements is $3 \times 2 \times 2 \times 1 \times 1 = 12$.

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6) If n is the number then $3n = \frac{3}{4} \times 120 = 90$. Then $n = 30$.

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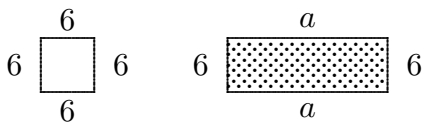
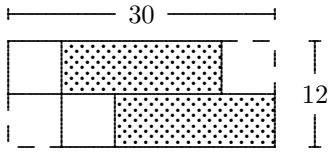
7) Each white square is 6×6; suppose that each shaded rectangle is $6 \times a$ as shown. Then each square has perimeter 24 and each shaded rectangle has perimeter $2a + 12$. Since the rectangles have double the perimeter of the squares, $2a + 12 = 2(24) = 48$. Solving for a gives $a = 18$. By applying this information to the original diagram, we see that perimeter of the original figure is 84.





8) Suppose there are n days in a week. Then there are $n \times n$ days in a month and $n \times n \times n$ days in a year. It follows that $n^3 = 1000$ and hence $n = 10$. There are 10 days in a week.

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9) After each cut, the amount of cake remaining is $\frac{2}{3}$ of what it was. After 3 cuts, the amount remaining is $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$ of the original cake.

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10) Suppose p is the original price. After applying the 10% discount, the price is $p - 0.10p = 0.90p$. After applying the tax, the price becomes

$$0.90p + (0.10)(0.90)p = 0.90p + 0.09p = 0.99p.$$

The final price is 0.99 or 99% of the original price.

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11) If we multiply $1 \times 3 \times 5 \times 7 \times 9 \times \cdots$ we obtain, in order, 1, 3, 15, 105, 945, ... In fact, since 5 is a factor in the overall product its last digit must be 0 or 5. Since the answer cannot be even, the last digit is 5.

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12) The last seat in Row 1 is number 24, the last one in Row 2 is number 48, the last one in Row 3 is number 72, and so on. It follows that the last seat in Row 15 is number 360, and so the 15th seat in Row 16 is number 375. The answer is Row 16.

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13) Since it is not possible for Juan to have a 0 for the hundreds digit, the smallest possible hundreds digit is 1. Starting with this, the smallest possible number Juan can obtain is 108.

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14) The powers of 2 are 2, 4, 8, 16, 32, 64, 128, 256, 512, etc. The last digits of these powers are 2, 4, 8, 6, 2, 4, 8, 6, 2, etc. The 2008th power of 2 will end in 6. After subtracting 2, the last digit of $2^{2008} - 2$ will be 4.

15) Since a is opposite an angle of $40°$, it is $40°$. Since s is the supplement for a $100°$ angle, it is $80°$. Since $a + s + y = 180°$, $y = 60°$. Since x and y are supplements, $x = 120°$.