

# THE UNIVERSITY OF THE WEST INDIES, MONA

Presents

## The 2012 Jamaican Mathematical Olympiad

### Test for Grades 9, 10, and 11

NAME: \_\_\_\_\_

GRADE: \_\_\_\_\_

SCHOOL: \_\_\_\_\_

PRINCIPAL: \_\_\_\_\_

YEAR OF BIRTH: \_\_\_\_\_

STUDENT PHONE: \_\_\_\_\_

CONTACT TEACHER: \_\_\_\_\_

CONTACT PHONE: \_\_\_\_\_

#### EXAMINATION QUESTIONS

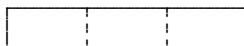
1) A rosebush in a garden has three branches. On each branch there is a cluster of three roses, and in each rose there are two bees. How many bees are in the rosebush?

- (a) 8                      (b) 18                      (c) 24                      (d) 27                      (e) 54

2) In a certain mathematics class, 17 students are in the Spanish class and 13 are in the French class. If four of them study both languages, how many study at least one foreign language?

- (a) 26                      (b) 17                      (c) 30                      (d) 34                      (e) 13

3) A square of area  $81 \text{ cm}^2$  is cut into three equal rectangles which are placed end-to-end, as shown below. What is the perimeter of the new rectangle?



- (a) 36 cm                      (b) 81 cm                      (c) 60 cm                      (d) 72 cm                      (e) 54 cm

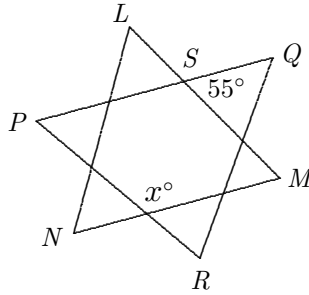
4) Marsha says that 25% of her books are novels and  $\frac{1}{9}$  of them are cookbooks. How many books does Marsha have if we know that she has between 50 and 100 books?

- (a) 93                      (b) 56                      (c) 64                      (d) 50                      (e) 72

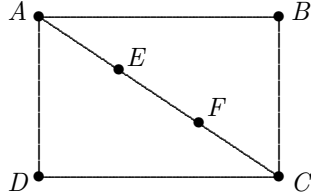
5) If  $4^x = 9$  and  $9^y = 256$  then  $xy$  is equal to:

- (a) 2                      (b) 4                      (c)  $-1$                       (d)  $\log_{36} 256$                       (e)  $\frac{1}{4}$

- 6) In the figure below, triangles  $PQR$  and  $LMN$  are equilateral. If  $\angle QSM$  is  $55^\circ$ , what is the value of  $x$ ?

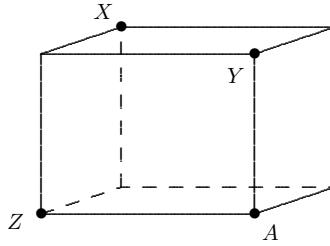


- (a)  $125^\circ$       (b)  $105^\circ$       (c)  $100^\circ$       (d)  $115^\circ$       (e)  $120^\circ$
- 7) The sum of four consecutive odd numbers is 48. What is the largest of these numbers?  
 (a) 13      (b) 15      (c) 17      (d) 19      (e) 21
- 8) What is the maximum number of digits a number can have if every pair of consecutive digits is a perfect square?  
 (a) 5      (b) 10      (c) 3      (d) 6      (e) 4
- 9) In the figure below,  $ABCD$  is a rectangle with  $AB = 5$  and  $AD = 3$ . The points  $E$  and  $F$  divide  $AC$  into three equal parts. What is the area of triangle  $BEF$ ?

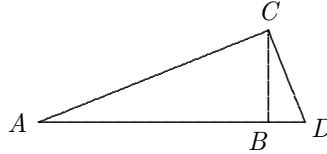


- (a)  $\frac{\sqrt{34}}{3}$       (b)  $\frac{5}{3}$       (c)  $\frac{\sqrt{68}}{3}$       (d)  $\frac{3}{2}$       (e)  $\frac{5}{2}$
- 10) When  $1000^{2012}$  is written as a numeral, how many digits does it have?  
 (a) 2013      (b) 6036      (c) 6037      (d) 8048      (e) 2016
- 11) Suppose  $a$  and  $b$  are positive real numbers greater than 1. Which of the following fractions has the greatest value?  
 (a)  $\frac{a}{b-1}$       (b)  $\frac{a}{b+1}$       (c)  $\frac{2a}{2b+1}$       (d)  $\frac{2a}{2b-1}$       (e)  $\frac{3a}{3b+1}$

- 12) The diagram below shows a cuboid with four of its vertices marked  $X$ ,  $Y$ ,  $Z$ , and  $A$ , respectively. If  $XY = 8$ ,  $XZ = \sqrt{55}$ , and  $YZ = 9$ , what is the length of  $XA$ ?

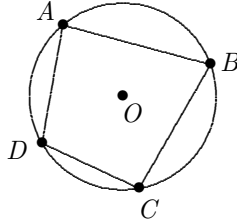


- (a)  $\sqrt{90}$       (b) 10      (c)  $10\sqrt{2}$       (d)  $\sqrt{120}$       (e) 11
- 13) When 1001 is divided by a certain 1-digit number the remainder is 5. What is the remainder when the same 1-digit number divides 2012?
- (a) 0      (b) 1      (c) 2      (d) 3      (e) 5
- 14) The difference  $\frac{a^2 - b^2}{ab} - \frac{ab - b^2}{ab - a^2}$  is equal to:
- (a)  $\frac{a}{b}$       (b)  $\frac{a^2 - 2b}{ab}$       (c)  $a^2$       (d)  $a - 2b$       (e)  $\frac{a^2}{b^2}$
- 15) In the figure below,  $ABC$  and  $ACD$  are right triangles with right angles at  $B$  and  $C$ , respectively. If  $AB = 64$  and  $AD = 100$ , what is  $AC$ ?

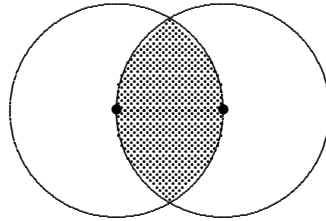


- (a)  $32\sqrt{3}$       (b) 72      (c) 80      (d) 84      (e)  $50\sqrt{3}$
- 16) The numbers 257 and 338 have the property that when their digits are put in reverse order the new numbers, 752 and 833 respectively, are larger. How many 3-digit numbers have this property?
- (a) 124      (b) 252      (c) 280      (d) 288      (e) 360
- 17) How many right triangles have one side of length  $\sqrt{60}$  and integer values for the lengths of the hypotenuse and the other side?
- (a) 0      (b) 1      (c) 2      (d) 3      (e) 6
- 18) Suppose  $M$  and  $N$  are positive real numbers with  $M \neq N$ . If  $\log_M N = \log_N M$ , what is  $MN$ ?
- (a)  $\frac{1}{2}$       (b) 1      (c) 2      (d) 10      (e)  $\frac{1}{10}$
- 19) The numbers  $a$ ,  $b$ , and  $c$  are prime numbers with  $a > b > c$ . If  $a + b + c = 78$  and  $a - b - c = 40$ , what is  $abc$ ?
- (a) 438      (b) 590      (c) 1062      (d) 1239      (e) 2006

- 20) In the figure below, the quadrilateral  $ABCD$  is inscribed in a circle with center  $O$ . It is known that  $\angle OAB = 35^\circ$ ,  $\angle OBC = 40^\circ$ , and  $\angle OCD = 55^\circ$ . What is  $\angle ODA$ ?



- (a)  $40^\circ$       (b)  $35^\circ$       (c)  $100^\circ$       (d)  $50^\circ$       (e)  $55^\circ$
- 21) Which of the following integers is odd?
- (a)  $(13 + 7)^5$       (b)  $5(7^2 - 13)$       (c)  $\frac{13}{7} + \frac{1}{2} - \frac{5}{14}$       (d)  $\frac{13 + 7^2}{2}$       (e)  $2(14 + 7)^{13}$
- 22) Let  $a$ ,  $b$ ,  $c$ , and  $d$  be real numbers such that  $|a - b| = 2$ ,  $|b - c| = 3$ , and  $|c - d| = 4$ . What is the sum of all the possible values of  $|a - d|$ ?
- (a) 18      (b) 9      (c) 15      (d) 24      (e) 12
- 23) In the figure below, two circles of radius 6 overlap in such a way that each passes through the centre of the other. What is the area of the shaded region?



- (a)  $24\pi$       (b)  $24\pi - 18\sqrt{3}$       (c)  $9\pi - 18$       (d)  $18\pi - 36$       (e)  $36\pi\sqrt{3}$
- 24) A number less than 3568 is odd, has remainder 2 when divided by 3, and has remainder 4 when divided by 5. What is the sum of the digits of the largest number which meets these conditions?
- (a) 14      (b) 17      (c) 18      (d) 20      (e) 21
- 25) What is the greatest number of elements that can be chosen from the set  $S = \{1, 2, 3, \dots, 24, 25\}$  so that the sum of any two elements is not divisible by 3?
- (a) 9      (b) 10      (c) 5      (d) 4      (e) 8

END OF QUESTIONS

You may mail your completed question paper to:

Mathematical Olympiad  
P.O. Box 94  
Mona Post Office  
Kingston 7

You may also deliver your entry by hand or by courier directly to the Department of Mathematics at the UWI, Mona. In all cases, an entry must be received by February 27, 2012 in order to be considered.

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<http://myspot.mona.uwi.edu/mathematics/> (see the link to the Olympiad Resource Centre).

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