THE UNIVERSITY OF THE WEST INDIES, MONA

Presents

The 2013 Jamaican Mathematical Olympiad

Test for Grades 9, 10, and 11

NAME: _____

GRADE: _____

SCHOOL: _____

PRINCIPAL: _____

YEAR OF BIRTH: _____

STUDENT PHONE: _____

CONTACT TEACHER: _____

CONTACT PHONE: _____

EXAMINATION QUESTIONS

1) Consider the following five expressions:

$$2-(-4); \qquad (-2)\times(-3); \qquad 2-8; \qquad 0-(-6); \qquad (-12)\div(-2)$$

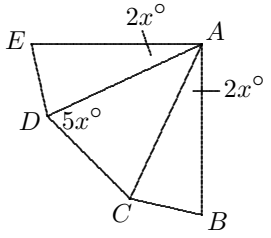
How many of them are *not* equal to 6?

(a) 0 (b) 1 (c) 2 (d) 4 (e) 5

2) What is the value of $(-1)^1+(-1)^2+(-1)^3+\cdots+(-1)^{2012}$?

(a) -2012 (b) -1 (c) 0 (d) 1 (e) 2012

3) In the figure below, $ABCDE$ is a pentagon with $AB=AC=AD=AE$. If $\angle EAB=90^\circ$, $\angle EAD=2x^\circ$, $\angle BAC=2x^\circ$, and $\angle ADC=5x^\circ$, what is the value of x?



(a) 15 (b) 12 (c) 14 (d) 10 (e) 20

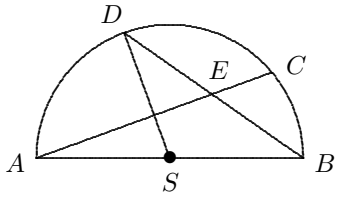
4) Western High School has 1,000 students and 570 of them are girls. One fourth of the students at Western High ride the bus to school, and exactly 313 of the boys do *not* ride the bus. How many girls ride the bus to school?

(a) 7 (b) 63 (c) 153 (d) 180 (e) 133

5) Two numbers P and Q are positive real numbers such that P is 40% larger than Q. What is the ratio $P : Q$?

(a) 40 : 1 (b) 5 : 7 (c) 7 : 5 (d) 5 : 2 (e) 5 : 3

6) In the figure below, $ADCB$ is a semicircle with centre S and diameter AB. The angle CAB is 20°. $DS \perp AC$, and E is the point where AC and DB intersect. What is the measure of $\angle AED$?



(a) 55° (b) 60° (c) 50° (d) 30° (e) 40°

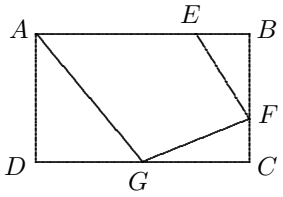
7) If $3^k = 9^{30}$, what is the value of k?

(a) 15 (b) 30 (c) 40 (d) 60 (e) 90

8) For how many prime numbers p is it true that $p^4 + 1$ is also prime?

(a) None (b) 1 (c) 2 (d) 3 (e) Infinitely many

9) In the figure below, $ABCD$ is a rectangle and E, F, and G are points lying on its sides such that DG is half of DC, CF is one-third of CB, and BE is one-fourth of BA. What is the ratio of the area of the quadrilateral $EFGA$ to the area of $ABCD$?



(a) $\frac{1}{2}$ (b) $\frac{7}{12}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$ (e) $\frac{3}{5}$

10) If $x^2yz^3 = 7^3$ and $xy^2 = 7^9$, What is xyz equal to?

(a) 7^4 (b) 7^6 (c) 7^8 (d) 7^9 (e) 7^{10}

11) At a book shop, each of the paperbacks cost \$500 and each of the hard-covered books cost \$700. Jordan spent exactly \$10,100 on books. What is the smallest number of books he could have bought?

(a) 19 (b) 18 (c) 17 (d) 16 (e) 15

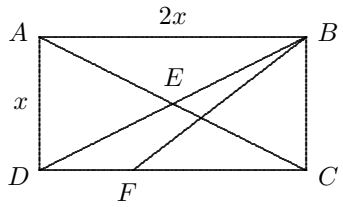
12) We say that a point (x, y) is a *grid point* to mean that at least one of its coordinates is an integer. How many grid point lie on the circle $x^2 + y^2 = 50$?

(a) 16 (b) 30 (c) 48 (d) 60 (e) 100

13) How many four-digit numbers are divisible by 3, have the digit 2 in the thousands place, and the digit 8 in the ones place?

(a) 34 (b) 30 (c) 19 (d) 20 (e) 33

14) In the figure below, $ABCD$ is a rectangle with $AB = 2x$ and $AD = x$. The diagonals AC and BD intersect at E. The point F on CD has the property that the area of triangle BFC is half the area of the pentagon $BAEDC$. What is the length of FC?

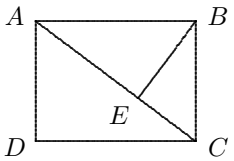


(a) $\frac{x}{2}$ (b) x (c) $\frac{5x}{4}$ (d) $\frac{3x}{2}$ (e) $\frac{3x}{4}$

15) Jacob had a pitcher of juice and poured 2/3 of it into a jar. From the jar, he drank 5/8 of the juice, poured 1/8 into a glass for later, and poured the rest of it back into the pitcher. What fraction of the original juice was in the pitcher then?

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{9}$ (d) $\frac{5}{4}$ (e) $\frac{1}{4}$

16) In the figure below, $ABCD$ is a rectangle with $AB = 4$ and $BC = 3$. The point E on AC has the property that EB is perpendicular to AC. What is the length of EB?



(a) 2.4 (b) 2.2 (c) 2.3 (d) 2.1 (e) 2.5

17) Mark has 10 cards. Each card is labeled with one of the following numbers: 3, 8, 13, 18, 23, 28, 33, 48, 53, 68. Each card has a different number. What is the minimum number of cards Mark has to choose so that the value of the cards totals exactly 100?

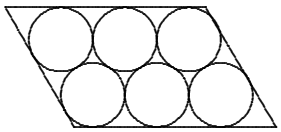
(a) 2 (b) 3 (c) 4 (d) 5 (e) Impossible

18) How many triples (a, b, c) of positive integers satisfy the simultaneous equations

$$\begin{cases} ab + bc = 44 \\ ac + bc = 23 \end{cases}$$

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

19) In the figure below, six circles which are tangent to their neighbors are inscribed in a parallelogram. If each circle has radius 3 cm, what is the area, in square centimetres, of the parallelogram?

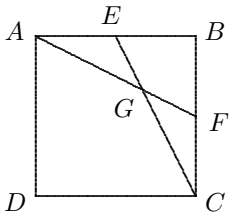


(a) 108 (b) $8(4 + 3\sqrt{3})$ (c) $15(2 + \sqrt{3})$ (d) $12(9 + 5\sqrt{3})$ (e) 216

20) If n is a positive integer we define $n! = 1 \times 2 \times 3 \times \cdots \times (n-1) \times n$. (The symbol $n!$ is called n *factorial*.) Suppose n is a positive integer and $n!$ has the prime factorization $2^{15} \times 3^6 \times 5^3 \times 7^2 \times 11 \times 13$. What is n?

(a) 15 (b) 14 (c) 13 (d) 17 (e) 16

21) In the figure below, $ABCD$ is a square, E and F are the midpoints of AB and BC, respectively, and G is the point where AF and EC intersect. What is the ratio of the area of the quadrilateral $AGCD$ to the area of $ABCD$?



(a) $\frac{1}{2}$ (b) $\frac{5}{8}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$ (e) $\frac{5}{9}$

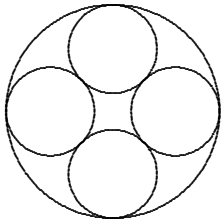
22) We say that a number is *primo* to mean any two consecutive digits in this number gives another number which is divisible by 17 or 23. How many numbers with 2012 digits are primo?

(a) 5 (b) 6 (c) 7 (d) 9 (e) More than 9

23) Suppose x, y, and z are real numbers with $x \neq 0$, $y \neq 0$, and $z \neq 0$. If $x + y + z = 5$, $xyz = 4$, and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2$, what is $x^2 + y^2 + z^2$ equal to?

(a) 9 (b) 4 (c) 25 (d) $\frac{25}{16}$ (e) 16

24) In the figure below four small, equal circles are circumscribed by a larger one. What is the ratio of the sum of the areas of the four smaller circles to the area of the larger one?



(a) $3 - 2\sqrt{2}$ (b) $2 - \sqrt{2}$ (c) $4(3 - 2\sqrt{2})$ (d) $\frac{1}{2}(3 - \sqrt{2})$ (e) $2\sqrt{2} - 2$

25) How many positive integers have the property that each of their digits is either 1 or 3 and the sum of their digits is 10?

(a) 34 (b) 28 (c) 35 (d) 56 (e) 55

END OF QUESTIONS

You may mail your completed question paper to:

Mathematical Olympiad
P.O. Box 94
Mona Post Office
Kingston 7

You may also deliver your entry by hand or by courier directly to the Department of Mathematics at the UWI, Mona Campus. In all cases, an entry must be received by December 10, 2012 in order to be considered.

For more information, a copy of this question paper, or the latest updates, please visit http://myspot.mona.uwi.edu/mathematics/ (see the link to the Olympiad Resource Centre).