

THE UNIVERSITY OF THE WEST INDIES, MONA CAMPUS

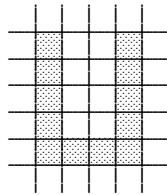
2014 JUNIOR MATHEMATICS OLYMPIAD

SOLUTIONS FOR GRADES 4, 5, AND 6

1. We will proceed from left to right. We have $0 + 1 = 1$, $1 + 2 = 3$, $3 + 3 = 6$, $6 + 4 = 10$, $10 - 3 = 7$, $7 - 2 = 5$, $5 - 1 = 4$, and $4 - 0 = 4$. The final answer is 4.

2. When Shanique bought her cookies she gave the salesperson \$100 and received \$10 in change. So, her cookies cost \$90 in all. Since the cookies cost \$30 apiece, she bought 3 cookies in all.

3. We can redraw the grid on top of the letter U. It is now clear that the U covered up 12 squares on the grid.



4. No friend who weighs 80 kg can go in the elevator with another friend who weighs 80 kg. Since there are three such friends, they must go up in three separate trips. However, these are all the trips that are needed. Indeed, two friends who weigh 80 kg and 60 kg each can make one trip. Then the other two friends can make two separate trips. They will need three trips in all.

5. We consider each expression in turn. First, $2 + 0 + 1 + 4 = 7$. Second, $201 - 4 = 197$. Third, $2 \times 0 \times 1 \times 4 = 0$. Fourth, $20 - 14 = 6$. Finally, $4 + 1 + 0 - 2 = 3$. The answers produced are 7, 197, 0, 6, and 3. The expression with the smallest value is $2 \times 0 \times 1 \times 4 = 0$.

6. Since one candy bar costs \$90, four of them cost \$360. But four candy bars plus three sweeties cost \$450. So, the three sweeties cost $\$450 - \$360 = \$90$. This means that each sweetie cost \$30.

7. Among the five numbers 1246, 3874, 4683, 4874, and 8462, Liana chose an even number. So, she did not choose 4683. She also did not choose a number with repeated digits. So, she did not choose 4874. The hundreds digits in her number was double the ones digit. So, she did not choose 1246. The two remaining numbers are 3874 and 8462. But the tens digit in Liana's number was greater than the thousands digit. So, she chose 3874.

8. We count the triangles first. There are 4 small (one-by-one) triangles in the diagram, as shown in Figure 8(a). There are 3 two-by-two triangles as shown in Figure 8(b). There are 2 three-by-three triangles

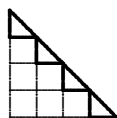


Fig. 8(a)

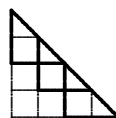


Fig. 8(b)

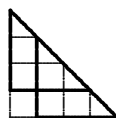


Fig. 8(c)

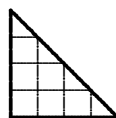


Fig. 8(d)

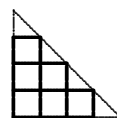


Fig. 8(e)

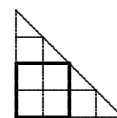


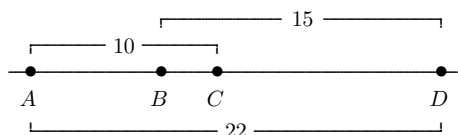
Fig. 8(f)

as shown in Figure 8(c). And there is 1 four-by-four triangle as shown in Figure 8(d). There are $4+3+2+1 = 10$ triangles in all. We now count the squares. There are 6 small (one-by-one) squares in the diagram, as shown in Figure 8(e). And there is 1 two-by-two square as shown in Figure 8(f). There are $6+1 = 7$ squares in all. Since $10 - 7 = 3$, there are 3 more triangles than squares in the figure.

9. There were 6 rows in front of Al, Bob, and Carl, one row consisting of Al, Bob, and Carl, and four rows behind them. There were $6 + 1 + 4 = 11$ rows of students in all. This means that 33 students went to the museum.

10. First, $7001 \times 100 = 701\,100$. Then $7001 \times 100 + 7001 = 700\,100 + 7\,001 = 707\,101$.

11. Let BC represent the distance from B to C . The distances from A to C and from B to D do not add up to 22, the distance from A to D . This is because they overlap between B and C . Since BC is counted twice, once in each interval, the distance from A to D is actually $10 + 15 - BC$. This is $25 - BC$, and it is equal to 22. So, $BC = 3$.

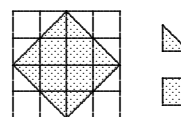


12. Al does not play volleyball or soccer because he does not like sports with a ball. He does not practice judo because Ben does. So, the statement “Al does karate” has to be true. The other four statements do not have to be true. Indeed, the statement “Al plays volleyball” is false (he does karate); the statement “Ben plays soccer” is false (he does judo); the statement “Dan does karate” is false (Al does karate). The other statement, “Carl plays volleyball” does not have to be true. Possibly, Al practices karate, Ben practices judo, Carl plays soccer, and Dan plays volleyball.

13. There had to be at least one boy and one girl at the picnic. However, the boy also had at least one brother there. So, there had to be at least two boys at the picnic. Also, the girl had at least one sister there. So, there had to be at least two girls at the picnic. There could have been two boys and two girls there, all brothers and sisters. Four is the smallest number of children that could have been at the picnic.

14. During one time period, Kaysia ate 2 bowls of ice cream and Daysia ate 3 bowls of ice cream. They ate 5 bowls in all. During the next time period, Kaysia ate 2 more bowls of ice cream and Daysia ate 3 more bowls of ice cream. They ate 5 bowls of ice cream during that time and 10 bowls in all. So, two time periods is one hour and each one is one half of an hour. During the first hour, Kaysia ate 4 bowls of ice cream (and Daysia ate 6).

15. The shaded region consists of 4 small squares and 8 small triangles. Each small square has an area of 1 cm^2 . Each small triangle is exactly one half of a square. So, its area is $\frac{1}{2}\text{ cm}^2$. The total shaded area, in cm^2 , is $(4 \times 1) + \left(8 \times \frac{1}{2}\right) = 4 + 4 = 8$.



16. There are 5 choices for the tens digit a : 1, 3, 5, 7, or 9. For each one, there are 4 choices for the ones digit b : it may be any odd digit except a . There are $5 \times 4 = 20$ such numbers in all. They are:

13	31	51	71	91
15	35	53	73	93
17	37	57	75	95
19	39	59	79	97

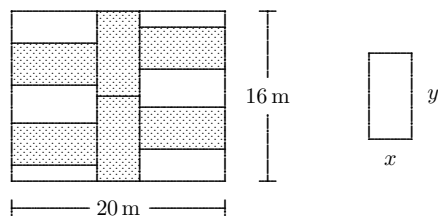
17. The first numbers Peter wrote were 3, 4, 5, 6, 7, 8, and 9. These there are 7 of these numbers in all, and they were the first 7 digits he wrote. The next numbers he wrote were 10, 11, 12, \dots , 19. There are 10 of these numbers in all, and they consisted of the next 20 digits Peter wrote. At this stage Peter had written $7 + 20 = 27$ digits in all. The next four numbers he wrote were 20, 21, 22, and 23. These four numbers have a total of 8 digits. At this stage, Peter had written $27 + 8 = 35$ digits in all. He stopped there, and the last whole number he wrote was 23.

18. Let x be teh number of birds ineach tree after some of them flew away. Before they flew away, there were $x + 6$ birds in the first tree, $x + 8$ birds in the second tree, and $x + 4$ birds in the third tree. So, there were

$$(x + 6) + (x + 8) + (x + 4) = (x + x + x) + (6 + 8 + 4) = 3x + 18$$

birds in the trees. This was equal to 60 birds. So, $3x + 18 = 60$. Subtracting 18 from both sides gives $3x = 42$. Dividing both sides by 3 gives $x = 14$. After the birds flew away, there were 14 birds in each tree. Before they flew away, there were $14 + 8 = 22$ birds in the second tree.

19. Let x be the width of a rectangle and let y be its height. From the middle two flowerbeds (which are up-and-down in the figure) $y + y = 16$ m. So, $y = 8$ m. From the width of the rectangular garden, we see that $y + x + y = 20$ m. This means that $8 \text{ m} + x + 8 \text{ m} = 20 \text{ m}$. So, $16 \text{ m} + x = 20 \text{ m}$. Subtracting 16 m from both sides gives $x = 4$ m. So, each flowerbed has width 4 m and height 8 m. The perineter of a flowerbed is $4 \text{ m} + 8 \text{ m} + 4 \text{ m} + 8 \text{ m} = 24 \text{ m}$.



20. Let x be the number Akeem chose. When he subtracted 203 from it, the result was $x - 203$. After he added 2003 to it the result was $x - 203 + 2003 = x + 1800$. This was equal to 20,003. So, $x + 1800 = 20003$. Subtracting 1800 from both sides gives $x = 18,203$.

21. It is not possible that $a = 0$. Otherwise, we would have $a + a + a = 0 + 0 + 0 = 0$. So, we would have $c = 0$. But a and c cannot be the same digit. Also, a cannot be 4 or more. Otherwise, $a + a + a$ would be 12 or more and so c would be 12 or more. But c is a digit and cannot be more than 9. So, the only possibilities are $a = 1, 2, \text{ or } 3$. In the same way, $b = 1, 2, \text{ or } 3$. Thus $c = 3, 6, \text{ or } 9$ and $d = 3, 6, \text{ or } 9$. Since c and d must be different digits, $c + d$ could be any of these expressions:

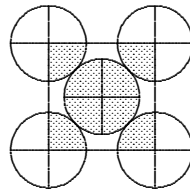
$$3 + 6; \quad 3 + 9; \quad 6 + 3; \quad 6 + 9; \quad 9 + 3; \quad 9 + 6$$

These sums are equal to e . They are 9, 12, 9, 15, 12, and 15, respectively. Since e is a single digit, the only possibility is $e = 9$. This happens when $c = 3$ and $d = 6$ or $c = 6$ and $d = 3$. In turn, this happens when $a = 1$ and $b = 2$ or $a = 2$ and $b = 1$.

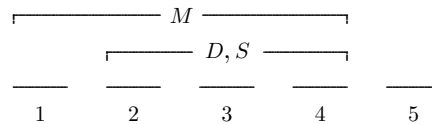
22. Pedro bought at least one small, one medium, and one large cookie, and bought 10 cookies in all. He could not have bought 4 or more large cookies. If he did, he would have paid at least 16 pesos for 4 large cookies, at least 2 pesos for 1 medium cookie, and at least 1 peso for the other 5 cookies. He would have spent at least $16 + 2 + 5 = 23$ pesos. Pedro could not have bought 3 large cookies either. If he did, he would have paid 12 pesos for the large cookies, at least 2 pesos for a medium one, and at least 1 peso for the other 6 cookies. He would have spent at least $12 + 2 + 6 = 20$ pesos. Pedro could not have bought 2 large cookies either. If he did, he would have paid 8 pesos for the large cookies, at least 2 pesos for a medium one, and at least 1 peso for the other 7 cookies. He would have spent at least $8 + 2 + 7 = 17$ pesos. So, Pedro bought 1 large cookie. He also bought 3 medium cookies and 6 small cookies. This would be 10 cookies in all and their cost in pesos would be $(1 \times 4) + (3 \times 2) + (6 \times 1) = 4 + 6 + 6 = 16$.

23. The bottles with capacity 18 oz., 24 oz., and 34 oz. are filled with orange juice and the bottles with capacity 16 oz. and 22 oz. are filled with cherry juice. In this case, there is a total of $18 \text{ oz} + 24 \text{ oz} + 34 \text{ oz} = 76 \text{ oz}$ of orange juice and $16 \text{ oz} + 22 \text{ oz} = 38 \text{ oz}$ of cherry juice. So, there is twice as much orange juice as cherry juice. The bottle with capacity 32oz. is empty.

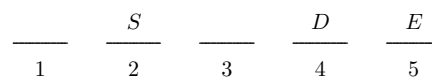
24. Divide each circle into four equal parts as shown in the figure below. The shaded regions consist of 8 equal quarter-circles. The unshaded regions consist of 12 equal quarter-circles. The ratio of the area of the shaded regions to the area of the unshaded regions is 8 : 12. This is equal to the ratio 2 : 3.



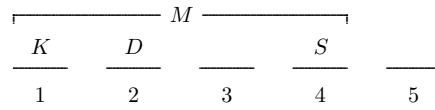
25. Let the seats be numbered 1, 2, 3, 4, and 5. Since Mary does not sit furthest to the right, she is in seat 1, 2, 3, or 4. Since Dorothy is not sitting furthest to the right, she is in seat 2, 3, 4, or 5. However, Ella is



sitting to the right of Dorothy. So Dorothy cannot be in seat 5; she is in seat 2, 3, or 4. Since Sylvia is not sitting furthest to the right or left, she is also in seat 2, 3, or 4. Finally, Sylvia is not sitting next to Dorothy. So, either Sylvia is in seat 2 and Dorothy is in seat 4, or *vice-versa*. Suppose Sylvia is in seat 2 and Dorothy



is in seat 4. We know that Ella is sitting to the right of Dorothy. So, Ella is in seat 5. But then Kathy would be sitting next to Sylvia, which she is not. So, Dorothy is in seat 2 and Sylvia is in seat 4. As before,



Mary is in one of the first four seats. Since Kathy is not sitting next to Sylvia, she is in seat 1. Then Mary is in seat 3, and Ella must be in seat 5. That is, Ella is sitting furthest to the right.