The University of the West Indies, Mona Campus

The 2014 Jamaican Mathematical Olympiad

First Round

Test for Grades 9, 10, and 11

Part A

This part consists of four multiple-choice questions. For each one, mark the letter for the correct answer ((a), (b), (c), (d), or (e)) in the answer book provided. Each question in this part is worth 5 marks.

1) Ava had a bag of sweeties. She gave half of them to Beth, Beth gave one-third of her share to Celine, and Celine gave one-fourth of her share to Davia. Davia received 3 sweeties from Celine. How many sweeties did Ava have in her bag?

(a) 72 (b) 64 (c) 108 (d) 84 (e) 12

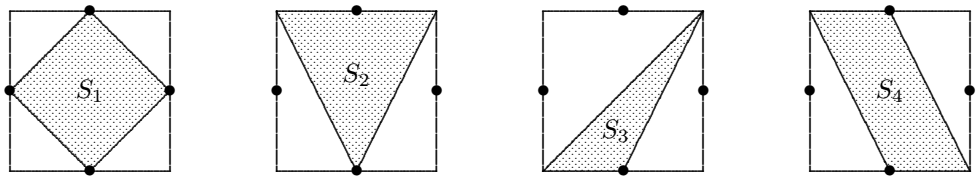
2) Which of the fractions below has the greatest value?

(a) $\frac{7}{8}$ (b) $\frac{66}{77}$ (c) $\frac{555}{666}$ (d) $\frac{4444}{5555}$ (e) $\frac{33333}{44444}$

3) Suppose m and n are integers and $P = 2^m$ and $Q = 3^n$. Which of the following expressions is equal to 12^{mn}?

(a) P^2Q (b) P^nQ^m (c) P^nQ^{2m} (d) $P^{2m}Q^n$ (e) $P^{2n}Q^m$

4) Each of the four squares in the figure below are congruent. In each of them the midpoints of the sides are marked with a solid dot and regions with areas S_1, S_2, S_3, and S_4, respectively, are shaded. Which expression below is true?



(a) $S_3 < S_4 < S_1 = S_2$
(b) $S_3 < S_1 = S_4 < S_2$
(c) $S_4 < S_3 < S_1 < S_2$
(d) $S_3 < S_1 = S_2 = S_4$
(e) $S_3 < S_4 < S_1 < S_2$

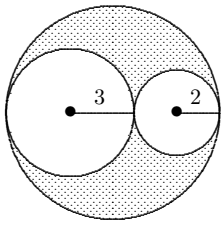
Part B

This part consists of eight written-answer questions. For each one, give your solution in the answer book provided. Each question in this part is worth 10 marks. To score full marks, you must provide an answer which is both correct and completely justified.

5) A grocer makes a display of cans in which the top row has one can and each lower row has two more cans than the row above it. Suppose the display contains 36 cans. How many rows does it contain?

6) For how many two-digit numbers ab is it true that a and b are odd numbers and $a \neq b$?

7) The figure below shows two circles with radii 3 and 2, respectively. They are tangent to each other and tangent to the larger circle around them both. What is the area of the shaded region?

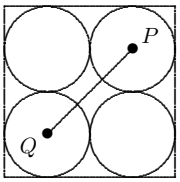


8) A $3 \times 3 \times 3$ cube is formed by gluing together 27 standard cubical dice. (On a standard die, the sum of the numbers on any pair of opposite sides is 7.) What is the smallest possible sum of all the numbers showing on the surface of the $3 \times 3 \times 3$ cube?

9) During a recent football match, the average age of the 11 players on the Strikers was 22. During the match one of them was injured and had to leave the field. The average age of the remaining players was 21. How old was the injured player?

10) Marcus has some \$10 and \$20 coins in a box. The number of \$10 coins is three times the number of \$20 coins. If he removes 8 coins of each type from the box, the number of \$10 coins will be five times the number of \$20 coins. What is the total value of the coins in his box?

11) In the figure below, four equal circles are inscribed in a square. The points P and Q are the centres of two of the circles. If each side of the square has length $2a$, what is the length of PQ?



12) What is $\sqrt[3]{p/q}$ if

$$p = (1)(2)(4) + (2)(4)(8) + (3)(6)(12) + \cdots + (2014)(4028)(8056)$$
$$q = (1)(3)(9) + (2)(6)(18) + (3)(9)(27) + \cdots + (2014)(6042)(18126)?$$