The University of the West Indies, Mona Campus

The 2014 Jamaican Mathematical Olympiad

First Round

Solutions for Grades 7 and 8

1. We have $2.6 + 0.12 = 2.60 + 0.12 = 2.72$.

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2. Let S be the number of sweeties Ava had in her bag. Beth received $\frac{1}{2}S = \frac{S}{2}$ sweeties from Ava. Celine received $\left(\frac{1}{3}\right)\left(\frac{S}{2}\right) = \frac{S}{6}$ sweeties from Beth. Davia received $\frac{1}{4}\left(\frac{S}{6}\right) = \frac{S}{24}$ sweeties from Celine. So, $\frac{S}{24} = 3$. Multiplying both sides by 24 gives $S = 72$. Ava had 72 sweeties in her bag.

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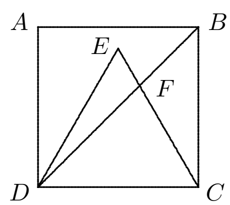
3. Since $ABCD$ is a square, $\angle DCB = 90°$. Also, $\triangle DCB$ is isosceles because $DC = CB$. Then $\angle CDB = \angle CBD$. Since the sum of the angles in $\triangle DCB$ is $180°$, $\angle CBD = \angle CDB = 45°$. Since



$\triangle DEC$ is equilateral, $\angle DCE = \angle DCF = 60°$. Thus $\angle FCB = \angle DCB - \angle DCF = 90° - 60° = 30°$. Finally, $\angle BFC + \angle FCB + \angle CBF = 180°$. Then $\angle BFC + 30° + 45° = 180°$ and so $\angle BFC + 75° = 180°$. Subtracting $75°$ from both sides gives $\angle BFC = 105°$.

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4. Let s be the number of stools and c the number of chairs in the room. We must count 5 legs for each stool and 6 legs for each chair (including the two legs for each child sitting on them). Then $5s + 6c = 39$. We must have $s = 1, 2, 3, \ldots,$ or 7. This is because if $s \geq 8$ then $5s$, the number of legs for the stools, would be 40 or more. This is impossible. So, we have $5s + 6c = 39$ with $s = 1, 2, \ldots, 7$. Solving for $6c = 39 - 5s$ and substituting $s = 1, 2, \ldots, 7$ into the right side, we obtain the respective equations

$$6c = 34, \quad 6c = 29, \quad 6c = 24, \quad 6c = 19, \quad 6c = 14, \quad 6c = 9, \quad \text{and} \quad 6c = 4.$$

The only equation with a whole-number solution for c is the third one, $6c = 24$. So, the unique solution is $c = 4$. There are 4 chairs in the room.

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5. Anna is 10 and her mother is four times as old as she is. So, Anna's mother is 40 years old. When Anna is twice as old as she is now, she will be 20 years old. This will be 10 years from now. Ten years from now, Anna's mother will be 50 years old.

6. Edward wrote down 10 numbers with the digit 7 in the ones place. These were 7, 17, 27, …, 97. So, he wrote 7 in the ones place 10 times in all. Edward also wrote down ten numbers with 7 in the tens place. These were 70, 71, 72, …, 79. So, he wrote 7 in the tens place 10 times in all. Edward wrote the digit 7 a total of $10 + 10 = 20$ times.

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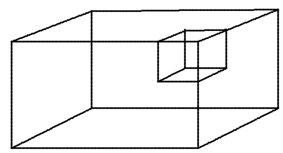
7. The volume of the original box is $15\,\text{cm} \times 10\,\text{cm} \times 8\,\text{cm} = 1200\,\text{cm}^3$. Eight corner cubes are removed. Each one has a volume of $3\,\text{cm} \times 3\,\text{cm} \times 3\,\text{cm} = 27\,\text{cm}^3$. So, the volume removed is



$8 \times 27\,\text{cm}^3 = 216\,\text{cm}^3$. As a proportion of the original volume, the volume removed is

$$\frac{216\,\text{cm}^3}{1200\,\text{cm}^3} = \frac{216}{1200} = \frac{18}{100} = 18\%$$

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8. Let x and y be the two numbers. So, $x + y = S$. If 3 is added to each of these numbers the results are $x + 3$ and $y + 3$, respectively. If both of the new numbers are doubled the results are $2(x + 3) = 2x + 6$ and $2(y + 3) = 2y + 6$, respectively. Finally, the sum of these last numbers is

$$(2x + 6) + (2y + 6) = 2x + 2y + 6 + 6 = 2(x + y) + 12 = 2S + 12.$$

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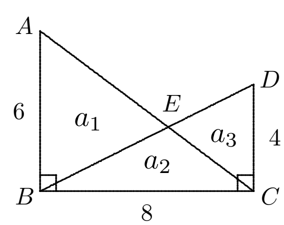
9. Let a_1 be the area of $\triangle ABE$, a_2 the area of $\triangle BEC$, and a_3 the area of $\triangle EDC$. The area of $\triangle ABC$ is $a_1 + a_2$. This is $\frac{1}{2}(6)(8) = 24$. The area of $\triangle BCD$ is $a_2 + a_3$. This is $\frac{1}{2}(8)(4) = 16$. The



difference of these areas is $24 - 16 = 8$. So, $(a_1 + a_2) - (a_2 + a_3) = 8$. That is, $a_1 - a_3 = 8$. The difference between the areas of $\triangle ABE$ and $\triangle EDC$ is $a_1 - a_3 = 8$.

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10. Let x be the weight in kilograms of the lightest pig. Then the weights of the seven pigs are x, $x + 5$, $x + 10$, …, $x + 30$. The sum of their weights is

$$\begin{aligned} &x + (x + 5) + (x + 10) + (x + 15) + (x + 20) + (x + 25) + (x + 30) \\ &\qquad = 7x + (5 + 10 + 15 + 20 + 25 + 30) = 7x + 105 \end{aligned}$$

So, $7x + 105 = 1001$. Subtracting 105 from both sides gives $7x = 896$. Dividing both sides by 7 gives $x = 128$. The lightest pig weighs 128 kg. The heaviest one weighs $x + 30 = 128 + 30 = 158$ kilograms.

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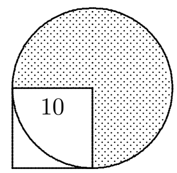
11. The union of the regions enclosed by the circle and the square may be divided into the region enclosed by the square and one enclosed by three-fourths of the circle (shaded in the figure below).



The area of the region enclosed by the square is $10\,\text{cm} \times 10\,\text{cm} = 100\,\text{cm}^2$. The area of the full circle, in cm^2, is $\pi(10)^2 = 100\pi$. The area of the shaded region, three-fourths of this, is $75\pi\,\text{cm}^2$. The area of the union of these regions is $100\,\text{cm}^2 + 75\pi\,\text{cm}^2 = (100 + 75\pi)\,\text{cm}^2$.

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12. In order for a number to be marvelous, its digits must be factors of 24. The digits which are factors of 24 are 1, 2, 3, 4, 6, and 8. Suppose a three-digit number is marvelous. If its largest digit is 8 then its other digits must be 3 and 1. There are 6 numbers of this type: 138, 183, 318, 381, 813, and 831. If its largest digit is 6 its other two digits could be 4 and 1 or 2 and 2. There are 6 numbers of the first type: 146, 164, 416, 461, 614, and 641. There are 3 numbers of the second type: 226, 262, and 622. If its largest digit is 4 its other digits must be 2 and 3. There are 6 numbers of this type: 236, 263, 326, 362, 623, and 632. The largest digit of a marvelous number cannot be 3, 2, or 1. In any of these cases, there is no pair of digits smaller or equal in value that give a product of 24. In all, there are $6 + 6 + 3 + 6 = 21$ such numbers in all.