THE UNIVERSITY OF THE WEST INDIES, MONA

The 2015 Jamaican Mathematical Olympiad

Solutions for Grades 7 and 8

1. The value of $2 \times 0 \times 1 \times 4$ is 0. So the value of $2014 - (2 \times 0 \times 1 \times 4)$ is $2014 - 0 = 2014$.

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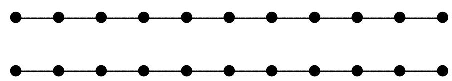
2. Since the path is 20 m long, 11 bushes can be planted on each side. (See the figure below.) A total of 22 bushes can be planted in this way.



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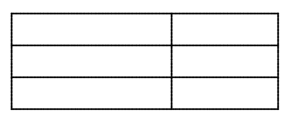
3. On the left side of the figure, there are 6 rectangles: three with a height of one small rectangle, two with a height of two small rectangles, and one with a height of three small rectangle. On the right side, there are 6 more rectangles. Also, there are 6 rectangles which span both sides of the figure. There are $6 + 6 + 6 = 18$ rectangles in all.



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4. The coins can be placed in the same box in three ways: both in the red box, both in the blue box, or both in the green box. They may be placed in different boxes in three ways: one in the red box and one in the blue, one in the red box and one in the green, or one in the blue box and one in the green. There are 6 ways of placing the coins in all.

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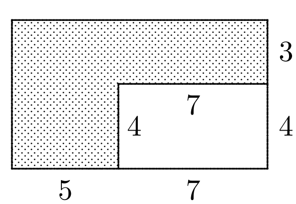
5. The six-sided polygon may be enclosed by a rectangle as shown below. The outer rectangle has width 12 and height 7. Its area is 84. The unshaded rectangle in the lower right corner has width 7 and height 4. Its area is 28. The area of the shaded polygon is $84 - 28 = 56$.



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6. After Eric left, there were 7 more boys than girls in Miss Smith's class. So, before he left there were 8 more boys than girls in the class. Since there are twice as many boys as girls, there are 16 boys and 8 girls in the class. After Eric and Joan left, there were 7 girls in the class.

7. Let c be the number of grandchildren that Grandmother has. If she gives each of them 2 dumplings, she will have 3 dumplings left over. Thus grandmother made $2c + 3$ dumplings. If she gives each of them 3 dumplings she will be 2 dumplings short. Thus Grandmother made $3c - 2$ dumplings as well. Then $2c + 3 = 3c - 2$. Adding 2 to both sides gives $2c + 5 = 3c$. Subtracting $2c$ from both sides gives $5 = c$. Grandmother has 5 children (and she made 13 dumplings for them).

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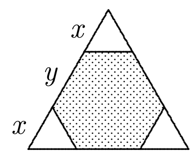
8. Let x be the side of a smaller triangle and y a longer side of the hexagon. Then $2x + y = 6$. (All dimensions in this solution are in cm.) Since the sum of the perimeters of the smaller triangles



equals the perimeter of the shaded hexagon, $9x = 3x + 3y$. Subtracting $3x$ from this equation gives $6x = 3y$. Dividing both sides by 3 gives $2x = y$. Substituting for y in the first equation gives $2x + 2x = 6$. Thus $4x = 6$ and hence $x = 6/4 = 3/2 = 1.5$.

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9. Suppose the time on a watch reads the same from the left as from the right. The times with the first digit 0 are 01:10, 02:20, 03:30, 04:40, and 05:50. There are five such times in this case. The times with the first digit 1 are 10:01, 11:11, 12:21, 13:31, 14:41, and 15:51. There are six such times in this case. The times with the first digit 2 are 20:02, 21:12, 22:22, and 23:32. There are four such times in this case. The total number of times reading the same from the left and the right is $5 + 6 + 4 = 15$.

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10. The boys were together again at the starting line when they each had run a whole number of laps in the same amount of time. Suppose Stephen ran s laps and Andrew ran a laps in this time. Stephen runs 1 lap in 12/5 minutes and so n laps in $12n/5$ minutes. Andrew runs 1 lap in 10/3 minutes and so m laps in $10m/3$ minutes. Since these times are equal, $12n/5 = 10a/3$. Cross multiplying gives $36n = 50a$ and hence $18n = 25a$. The smallest value of n such that $18n$ is a multiple of 25 is $n = 25$. In this case, $a = 18$ as well. Then the number of laps they ran together was $n + a = 25 + 18 = 43$.

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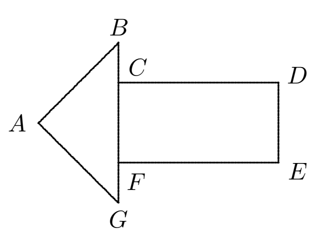
11. From the information given, $CDEF$ is a rectangle. Then $CF = DE = 10$ and so $BG = 20$. The $\triangle ABG$ has a right angle at A and $AB = AG$. It follows that $\angle ABG = \angle AGB = 45°$. By the Pythagorean theorem,



$(AB)^2 + (AG)^2 = (BG)^2 = 20^2 = 400$. Since $AG = AB$, $2(AB)^2 = 400$. Dividing both sides by 2 gives $(AB)^2 = 200$. Triangle BAG has base AG and perpendicular height AB. Its area is

$$\frac{1}{2}(AB)(AG) = \frac{1}{2}(AB)^2 = \frac{1}{2}(200) = 100.$$

The area of $CDEF$ is $(20)(10) = 200$. The area of $ABCDEFG$ is $100 + 200 = 300$.

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12. When 999 is divided by n, the remainder is 3. This means that 996 is divisible by n evenly. Thus $996 = kn$ for some positive integer k. Multiplying both sides by 2 gives $1992 = (2k)n$. Adding 9 to both sides gives $2001 = (2k)n + 9$. Then when 2001 is divided by n the remainder is 9.

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13. Let j be the capacity of a carton of juice and g the capacity of a glass. Then $\frac{3}{4}j = \frac{3}{2}g$. Multiplying both sides by $4/3$ gives $j = 2g$. That is, one carton of juice fills 2 glasses evenly. Then 5 cartons fills 10 glasses evenly.

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14. Let w be the weight of Carl the Camel when he is thirsty. After he drinks, he weighs 800 kg and water makes up 85% of his weight. After he drinks, Carl consists of 120 kg of solid matter and 680 kg of water. Before he drinks, he consists of 120 kg of solid matter and the rest of him, 84%, is water. So, before he drinks he consists of 120 kg of solid matter and this is 16% of his weight. Then

$$\frac{120}{w} = \frac{16}{100} = \frac{4}{25}.$$

Cross multiplying gives $3000 = 4w$. Dividing both sides by 4 gives $w = 750$.

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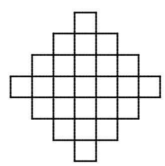
15. The original polygon has 28 equal sides and a perimeter of 56. So, each side has length 2. The interior of the polygon contains 25 equal squares (see the figure below). Each square has a side length of 2 and an area of 4. The area of the original polygon is $25 \times 4 = 100$.



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16. Mike typed two 7's, and the digits 2, 0, 1, and 4 (in that order), The first 7 he typed could have been in 5 different places. He could have typed 7xxxxx, 27xxxx, 207xxx, 2017xx, or 20147x. In the first case, there are 5 places where the second 7 could have been. In the second case, there are 4 places it could have been. In the third case there are 3 places, in the fourth case there are 2 places, and in the last case there is one place the second 7 could be. There are $5+4+3+2+1 = 15$ different numbers Mike could have typed.

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17. The smallest of the five numbers Carlos added was 14 or less. (If it were 15 or more the total would be at least 75.) Furthermore, the smallest number could be 14. For example, the numbers he added could be 14, 14, 14, 15, and 15. In this case, the sum of the five numbers minus the smallest one is $72 - 14 = 58$. If the smallest number is less than 14 then 72 minus the smallest number will be more than 58. So, the smallest final result Carlos can obtain is 58.

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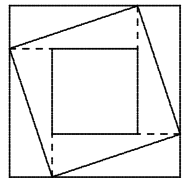
18. The square in the middle consists of the inner square plus four right-angled triangles. The area of the inner square is $4\,\text{cm}^2$ and the area of the outer square is $16\,\text{cm}^2$. So, the area of the section



in between, consisting of 8 equal triangles, is $16\,\text{cm}^2 - 4\,\text{cm}^2 = 12\,\text{cm}^2$. The area of each triangle is $12\,\text{cm}^2/8 = 1.5\,\text{cm}^2$. The area of four triangles is $6\,\text{cm}^2$. So, the area of the middle square is $4\,\text{cm}^2 + 6\,\text{cm}^2 = 10\,\text{cm}^2$.

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19. Let the original number be $ab2$, where $a, b \in \{0, 1, 2, \ldots, 9\}$. Then the new number is $2ab$. The value of the first number is $a \times 100 + b \times 10 + 2 = 100a + 10b + 2$. The value of the second one is $2 \times 100 + a \times 10 + b = 200 + 10a + b$. Since the first number is equal to the second one plus 36,

$$(100a + 10b + 2) = (200 + 10a + b) + 36 = 236 + 10a + b.$$

Subtracting $10a + b + 2$ from both sides gives $90a + 9b = 234$. Dividing throughout by 9 gives $10a + b = 26$. Then $a = 2$ and $b = 6$. The original number is 262. The sum of its digits is $2 + 6 + 2 = 10$.

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20. The prime factorization of 1664 is $2^7 \times 13$. So, the age of one of Mr. Williams's children is 13 or a multiple of 13, and the ages of his other children are powers of 2. It is not possible that the youngest child is 13 or a multiple of 13. Otherwise, the oldest child would also be a multiple of 13 and the product of their ages would have a factors of 13^2. Similarly, the age of the oldest child could not be a multiple of 13. (Otherwise, the younger one would also have to be a multiple of 13.) Then Mr. Williams has at least three children and one of them (neither the oldest nor the youngest) is 13 or a multiple of 13. Since the age of the oldest child is a power of 2, he or she is at least 16. The youngest child is at least 8, and one child in between is at least 13. The product $8 \times 13 \times 16$ is equal to 1664. If Mr. Williams had more than three children, or the oldest one is more than 16, then the product of their ages would be more than 1664. Since this is impossible, Mr. Williams has three children (and their ages are 8, 13, and 16).

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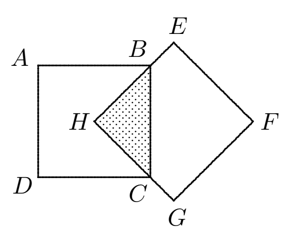
21. The polygon $ABEFGCD$ is the union of the two overlapping squares. Its area is the sum of the areas of the two squares minus the area of the overlapping part (shaded in the diagram below). Each square has an area of 100. The overlapping region is exactly one-fourth of the square $ABCD$. Its area is 25. Then the area of the polygon $ABEFGCD$ is $100 + 100 - 25 = 175$ square units.



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22. The boy did not give the same answer on two consecutive days. So, he did not give an answer on both Thursday and Friday. If he missed Thursday then he started on Friday and went until

Wednesday. If he missed Friday then he started on Saturday and went until Thursday. In the first case, he would have said "John" on Friday and then "Peter" on Tuesday. This is possible provided his name is John. In the second case, he would have said "John" on Saturday, "Bob" on Tuesday, and "Bob" on Thursday. This is impossible because he lies on Tuesdays and tells the truth on Thursdays. So, He missed Thursday. He started giving his name on a Friday and said "John". On the seventh day, a Thursday, he told the truth and said "John".

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23. Let s be the sum of the four numbers Nick crossed out. The sum of the numbers Pete crossed out is $3s$. The sum of all eight numbers they crossed out is $s + 3s = 4s$. This is clearly a multiple

4	12	8
13	24	14
7	5	23

of 4. The sum of the original nine numbers is $4+12+8+13+24+14+7+5+23 = 110$. This has a remainder of 2 when divided by 4. Since the eight numbers crossed out add up to a 4-multiple, the number not crossed out has a remainder of 2 when it is divided by 4. Among the nine numbers in the square, 14 is the only one with a remainder of 2 when divided by 4. So, 14 is be the number that neither Nick nor Pete crossed out. (One can also find by trial and error that Nick crossed out 4, 5, 7, and 8, and Pete crossed out 12, 13, 23, and 24.)

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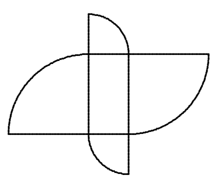
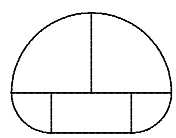
24. The perimeter of Emily's bird consists of two quarters of a large circle, two quarters of a small circle, two segments equal to a long side of the rectangle, and two segments equal to a short side.



Emily's Bird



Benjamin's Egg

The perimeter of Benjamin's egg consists of two quarters of a large circle, two quarters of a small circle, and one long side of the rectangle. The perimeter of the bird is larger by an amount equal to one long and two short sides of the rectangle. This is $10\,\text{cm} + 2(5\,\text{cm}) = 10\,\text{cm} + 10\,\text{cm} = 20\,\text{cm}$.

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25. Let r be Peter's original speed, in k/h, let t be the number of hours it takes to travel from P to Q, and let d the number of kilometres from P to Q. Since rate times time equals distance, $rt = d$. If Peter increases his speed by 3 k/h, he needs only one-third of the time to get from P to Q. So,

$$(r+3)\frac{t}{3} = d, \quad \text{and hence} \quad \frac{rt}{3} + t = d.$$

Multiplying the second equation by 3 gives $rt + 3t = 3d$. Multiplying the original equation, $rt = d$, by 3 gives $3rt = 3d$. Eliminating $3d$ from these equations gives $3rt = rt + 3t$. Dividing throughout by t gives $3r = r + 3$. Subtracting r from both sides gives $2r = 3$. Thus $r = 3/2$. (This is measured in k/h.) If Peter increases his speed by 6 k/h. his new speed will be $6 + \frac{3}{2} = 15/2$. This is five times faster then his original speed. So, if he increases his speed by 6 k/h he will arrive 5 times faster.