

The 2015 Jamaican Mathematical Olympiad

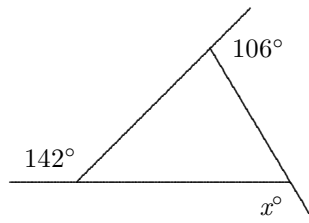
Practice Problem Set 2

- 1) In which of the following numbers is the product of its digits greater than the sum of its digits?
(a) 112 (b) 209 (c) 312 (d) 222 (e) 211

2) What is the value of $1 - 2 + 3 - 4 + 5 - 6 + \dots + 53 - 54 + 55$?

3) Mother gave each of her children one-twelfth of a pizza. Then she ate the remaining one-third of the pizza. How many children does Mother have?

4) In the figure below, the sides of a triangle have been extended to form three angles as shown. What is the value of x ?

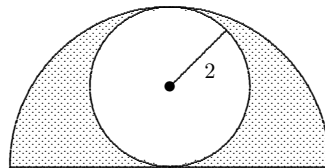


5) Which four digits need to be removed from the number 4921508 to get the smallest possible three-digit number? (When the digits are removed, the ones remaining stay in their original order.)

6) In a certain mathematics class, 17 students are in the Spanish class and 13 are in the French class. If four of them study both languages, how many study at least one foreign language?

7) The positive integer n is the product of all the prime numbers less than 2014. What is the ones digit of n ?

8) In the figure below, a circle of radius 2 is inscribed in a semicircle. What proportion of the area of the semicircle is shaded?



9) How many times between 00:00 and 23:59 does an electronic watch show all of the following four digits, 2, 0, 1, 2, in any order?

10) Three lights are flashing at regular intervals. One flashes every 2 minutes, one every 2.5 minutes, and one every 3 minutes. If all three lights flash at exactly 9:00 am, when is the next time they will all flash together?

- 11) The first four figures in a patterned tile arrangement are shown below. Figure 1 consists of 1 tile and Figure 3 consists of 7 tiles (including squares of various sizes). How many tiles will the sixth figure have?

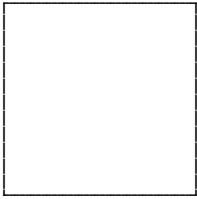


Figure 1

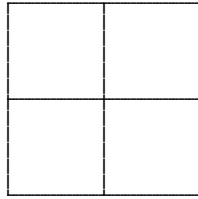


Figure 2

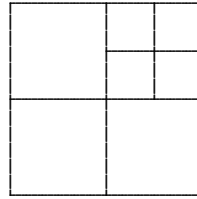


Figure 3

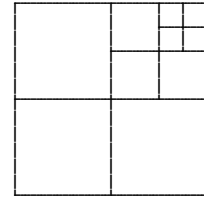
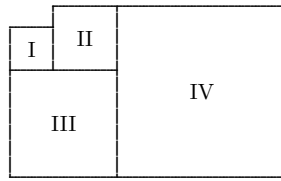


Figure 4

12) Find the value of $1 + \frac{1}{1 + \frac{1}{1+1}}$.

- 13) In the figure below, Regions I, II, III, and IV are squares. The perimeter of Square I is 16 cm and the perimeter of Square II is 24 cm. What is the perimeter, in centimetres, of Square IV?



- 14) The value of the two-digit number AB is equal to $5(A + B)$. What is the number AB ? [Note: In the expression AB , A is the tens digit and B is the ones digit.]

- 15) A *multiplication magic square* has the property that the product of the three numbers in each row, column, and diagonal is equal to 1. For the multiplication magic square below, what is the value of $r + s$?

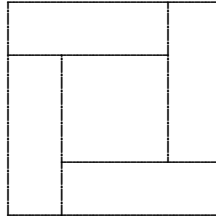
p	q	r
s	1	t
u	4	$1/8$

- 16) What is the last digit of the number $2^{2014} + 3^{2014}$?

- 17) Determine the number of three-digit numbers in which the third digit (the ones digit) is larger than the first digit (the hundreds digit).

- 18) When 1001 is divided by a certain one-digit number the remainder is 5. What is the remainder when the same one-digit number divides 2014?

- 19) In the figure below, four congruent rectangles are arranged to form a square as shown. If the area of the outer square is four times the area of the inner square, what is the ratio of the length of a long side of a rectangle to the length of a shorter side?

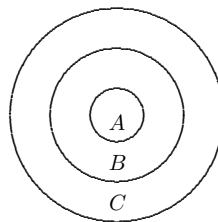


- 20) A vendor sells 360 chocolates each week. She purchases them at a supermarket at a cost of \$300 for a box of 8. However, she discovered that she can get a case of 60 at a wholesale shop for \$2,000. If she buys her chocolates from the wholesale shop, how much will she save each week?
- 21) Three married couples are attending a party. In how many ways can they form a three-person group in which there will not be a married couple?

- 22) The difference $\frac{a^2 - b^2}{ab} - \frac{ab - b^2}{ab - a^2}$ is equal to:

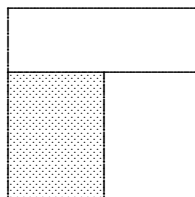
(a) $\frac{a}{b}$ (b) $\frac{a^2 - 2b}{ab}$ (c) a^2 (d) $a - 2b$ (e) $\frac{a^2}{b^2}$

- 23) Anna started throwing darts at the dart board shown below. She scored A points for each dart in the innermost circle, B for each one in the middle ring, and C for each one in the outer ring. Each turn,

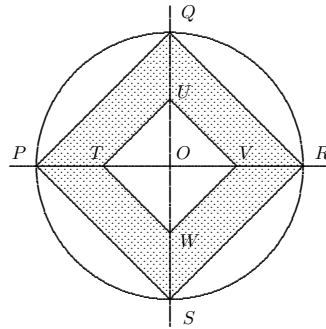


she threw three darts. On her first turn two darts landed in ring B and one in ring C , and she scored 10 points. In the second turn two darts landed in circle A and one in ring C , and she scored 22 points. On her next turn one dart landed in each of the three regions. How many points did she score?

- 24) In the figure below, a square with side length 12 cm has been divided into three rectangles with equal perimeter. What is the area of the shaded rectangle?



- 25) Mr. and Mrs. Dobson have several children. The average age of the Dobson family is 18 years. Without the 38-year-old father, the average age of the family decreases to only 14 years. How many children do the Dobsons have?
- 26) How many positive integers less than 1,000 are neither multiples of 5 nor of 7?
- 27) In the figure below, the circle $PQRS$ has centre O and radius 4, and PR and QS are perpendicular diameters of the circle. The points $T, U, V,$ and W are the midpoints of $PO, QO, RO,$ and $SO,$ respectively. What proportion of the area of the circle is shaded?



- 28) How many right triangles have one side of length $\sqrt{60}$ and integer values for the lengths of the hypotenuse and the other side?
- 29) The numbers 1, 2, and 3 are written on the circumference of a circle (see Figure 1). Then the sum of each pair of neighboring numbers is written between them (see Figure 2). In this way, 6 numbers are

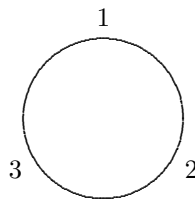


Fig. 1

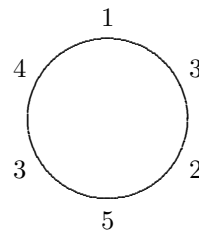


Fig. 2

arranged around the circle: 1, 3, 2, 5, 3, and 4. The process of adding each pair of neighboring numbers and placing their sum between them is repeated 3 more times, resulting in 48 numbers on the circle. What is the sum of these numbers?

- 30) In the figure below, two circles of radius 6 overlap in such a way that each passes through the centre of the other. What is the area of the shaded region?

