The 2015 Jamaican Mathematical Olympiad

Practice Problem Set 7

1) In the table below, the letters A, B, C, ..., G represent numbers (not necessarily distinct). If the sum of any three consecutive numbers is 15, what is the value of F?

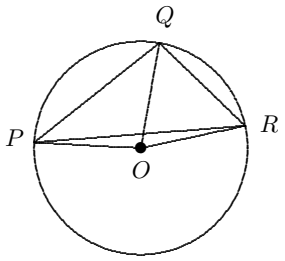
3	A	B	C	D	8	E	F	G

2) The positive integers greater than 1 are arranged in 5 columns as shown below:

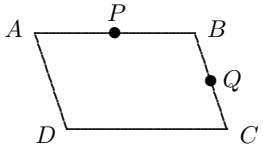
	2	3	4	5
9	8	7	6	
	10	11	12	13
17	16	15	14	
⋮	⋮	⋮	⋮	⋮

In which column will 1,000 be located?

3) In the figure below, P, Q, and R are points on a circle with centre O. If $\angle OPR = 5°$ and $\angle OQP = 40°$, what is $\angle OQR$?



4) In parallelogram $ABCD$ below, P is the midpoint of AB and Q is the midpoint of BC. If the area of $ABCD$ is 24, what is the area of $\triangle DPQ$?

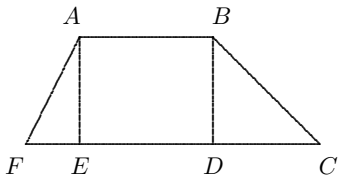


5) A train consists of an engine and five cars: I, II, III, IV and V. In how many ways can the cars be arranged so that car I is closer to the engine than car II?

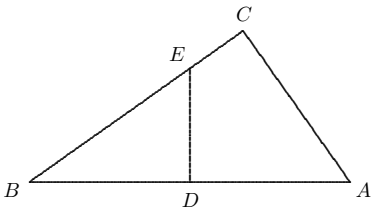
6) Maria has two apples, two bananas, and one mango. Each school day, from Monday to Friday, she will eat one piece of fruit. In how many ways can she do this?

7) Find all pairs of positive integers (x, y) such that $2^{2x} - 3^{2y} = 55$.

8) In the figure below, $ABDE$ is a rectangle and the points C, D, E, and F are collinear. If the rectangle $ABDE$ has area 80 and the trapezoid $ABCF$ has area 128, what is $AB : CF$?

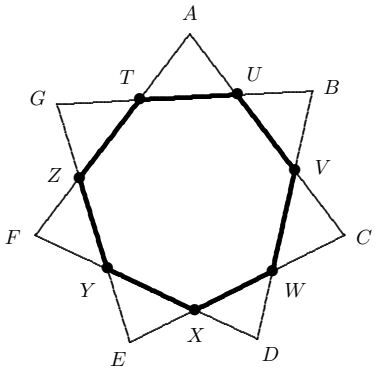


9) In the figure below, $\angle C = 90°$, $AD = DB$, $DE \perp AB$, $AB = 20$, and $AC = 12$. What is the area of quadrilateral $ADEC$?



10) Let $S = \{1, 2, 3, \ldots, 24, 25\}$. The set T is a subset of S with the property that the sum of any two elements in T is not divisible by 3. What is the largest number of elements that T could have?

11) In the figure below, the sides of the heptagon $TUVWXYZ$ (which is not necessarily regular) have been extended to form a "7-pointed star". Find the sum of the angles at A, B, C, D, E, F, and G.



12) We wish to tile a patio with sides of length positive integer N. We have two types of tiles: 5×5 squares and 1×3 rectangles. Determine the values of N for which this is possible. (The patio must be completely tiled with no tiles overlapping.)