

The University of the West Indies, Mona Campus
The 2015 Jamaican Mathematical Olympiad

FIRST ROUND EXAMINATION, GRADES 7 AND 8

JANUARY 17, 2015, AT 12:30 PM

Part A

This part consists of four multiple-choice questions. For each one, mark the letter for the correct answer ((a), (b), (c), (d), or (e)) on Page 3 of the answer book provided. Each question in this part is worth 5 marks.

1) How many hours are there in one-half of one-third of a quarter of a day?

- a) $\frac{1}{3}$ b) $\frac{1}{2}$ c) 1 d) 2 e) 3

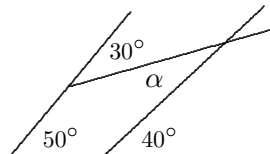
2) Alicia is packing 178 red and 121 blue blocks into boxes. Each box can hold no more than 10 blocks, and all blocks in the same box must have the same colour. What is the fewest number of boxes Alicia will need?

- a) 13 b) 18 c) 24 d) 30 e) 31

3) At the beginning of the game, John, Peter, and Karl had marbles in the ratio of 1 : 2 : 3. At the end of the game, the marbles were divided between them in the ratio of 4 : 5 : 6. What was the result of the game?

- a) John and Peter lost, and Karl won.
b) John and Karl won, and Peter lost.
c) John lost, Karl won, and Peter kept the same number of marbles.
d) John won, Karl lost, and Peter kept the same number of marbles.
e) None of the situations listed above took place.

4) In the figure below, what is the measure of angle α ?

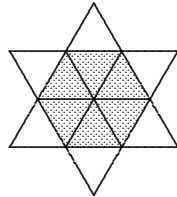


- a) 20° b) 25° c) 30° d) 35° e) 40°

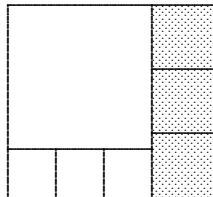
Part B

This part consists of six written-answer questions. For each one, give your solution in the answer book provided. Each question in this part is worth 10 marks. To score full marks, you must provide an answer which is both correct and completely justified.

- 5) Carlos opened his dictionary and said, “If I add the number of the page I am looking for to the number of the next page, I will get 341”. What page is Carlos looking for?
- 6) Let $s = 2012 + 2013 + 2014 + 2015 + 2016$. If s is divided by 2015, what will the remainder be?
- 7) The star in the figure below is constructed from 12 identical equilateral triangles and has a perimeter of 36 cm. What is the perimeter of the shaded hexagon?



- 8) An empty truck weighs 2,000 kg. After the truck was loaded, the freight (that is, the load) made up 80% of the weight of the loaded truck. At the first stop, one-fourth of the freight was unloaded. What percent of the loaded truck’s weight was the freight after that?
- 9) The rectangle below has been divided into 7 squares. Each shaded square has a side length of 8 cm. What is the side length of the largest unshaded square?



- 10) Let n be the smallest natural number such that the digits of n add up to 2015. What is the first digit of n ? (The first digit is the one furthest to the left.)

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FIRST ROUND SOLUTIONS, GRADES 7 AND 8

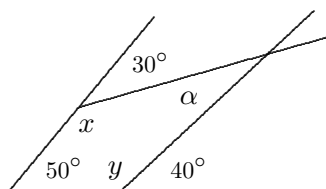
JANUARY 17, 2015, AT 12:30 PM

1. One day consists of 24 hours. A quarter of a day consists of 6 hours. One-third of a quarter of a day consists of one-third of 6 hours, which is 2 hours. One-half of one-third of a quarter of a day consists of one-half of 2 hours. This is 1 hour.

2. To pack the red blocks, Alicia needs at least 18 boxes. This is because 17 boxes can hold only 170 blocks, but 18 or more boxes can hold 180 or more blocks. Similarly, to pack the blue blocks she needs 13 or more boxes. She needs at least $18 + 13 = 31$ boxes in all.

3. At the beginning of the game, John had $1/6$ of the marbles, Peter had $2/6 = 1/3$ of the marbles, and Karl had $3/6 = 1/2$ of the marbles. At the end of the game, John had $4/15$ of the marbles, Peter had $5/15 = 1/3$ of the marbles, and Karl had $6/15 = 2/5$ of the marbles. John won because he went from $1/6$ to $4/15$ of the marbles. (Using a common denominator of 30, he went from $5/30$ to $8/30$ of the marbles.) Peter kept the same number of marbles because he started and finished with $1/3$ of them. Karl lost because he went from $1/2$ to $6/15$ of the marbles. (Using a common denominator of 30, he went from $15/30$ to $12/30$ of the marbles.) So, John won, Karl lost, and Peter kept the same number of marbles.

4. Let x and y be the angles shown below. Since x is the supplement of a 30° angle, $x = 180^\circ - 30^\circ = 150^\circ$. Similarly, $y = 180^\circ - 40^\circ = 140^\circ$. The sum of the angles in any quadrilateral is 360° . Thus $150^\circ + 50^\circ + 140^\circ + \alpha = 360^\circ$. That is, $340^\circ + \alpha = 360^\circ$. Then $\alpha = 20^\circ$



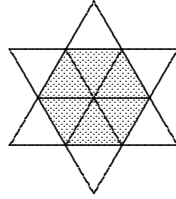
5. Let p be the number of the page Carlos is looking for. The number of the next page is $p + 1$. The sum of these page numbers is $p + (p + 1) = 2p + 1$. Equating this to 341 gives $2p + 1 = 341$. Subtracting 1 from both sides gives $2p = 340$. Dividing both sides by 2 gives $p = 170$. Carlos is looking for page 170.

6. First, $2014 + 2015 + 2016 = (2015 - 1) + 2015 + (2015 + 1) = 2015 + 2015 + 2015 = 3(2015)$. Second, $2012 + 2013 = (2010 + 2) + 2013 = 2010 + (2 + 2013) = 2010 + 2015$. then

$$s = (2012 + 2013) + (2014 + 2015 + 2016) = (2010 + 2015) + 3(2015) = 4(2015) + 2010.$$

If s is divided by 2015 the quotient will be 4 and the remainder will be 2010.

7. The perimeter of the star consists of 12 equal segments and their total length is 36 cm. So, each segment has a length of 3 cm. This means that each equilateral triangle has a side length of 3 cm. The perimeter of the hexagon consists of 6 of these sides. The perimeter of the hexagon is $6 \times 3 \text{ cm} = 18 \text{ cm}$.



8. After the truck was loaded, the freight made up 80% of the total weight. So, the truck itself was 20% of the total weight. Let w be the total weight (measured in kg) of the truck and the freight. Then

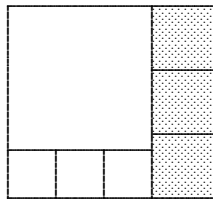
$$\frac{2000}{w} = \frac{20}{100} = \frac{1}{5}$$

Cross-multiplying gives $w = 10,000 \text{ kg}$. Then the truck weighs 2,000 kg and the freight weighs 8,000 kg. At the first stop, one-fourth of the freight was unloaded. This means that 2,000 kg was unloaded and 6,000 kg remained on the truck. The combined weight of the truck and freight was then 8,000 kg. The weight of the freight compared to the total weight was

$$\frac{6000}{8000} = \frac{3}{4} = \frac{75}{100}$$

The freight made up 75% of the total weight.

9. Let x be the side length of the largest unshaded square. It is clear from the diagram that the side length of the smaller unshaded squares is $x/3$. Also, the side length of each shaded square is 8. The



height of the outer rectangle is equal to the combined height of the three shaded squares. This is 24 cm. This height is also equal to the sum of the heights of the large unshaded square and one small unshaded square. This is $x + x/3 = 4x/3$. Then $4x/3 = 24$. Multiplying both sides by 3 gives $4x = 72$. dividing both sides by 4 gives $x = 18$. The side length of the largest unshaded square is 18 cm.

10. If the numeral for a certain positive integer has more digits than the numeral for another one, the number with more digits is the larger number. Also, the largest digit is 9. When 9 is divided into 2015 the quotient is 223 and the remainder is 8. So, any integer whose digits add up to 2015 must have at least 224 digits. Furthermore, if the number has 224 digits then 1 of them must be an 8 and the other 223 must be a 9. The smallest such number is 89,999, . . . ,999. The first digit of this number is 8.