The University of the West Indies, Mona Campus

The 2015 Jamaican Mathematical Olympiad

First Round Examination, Grades 9, 10, and 11

January 17, 2015, at 9:30 am

Part A

This part consists of four multiple-choice questions. For each one, mark the letter for the correct answer ((a), (b), (c), (d), or (e)) on Page 3 of the answer book provided. Each question in this part is worth 5 marks.

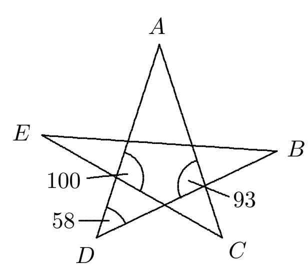
1) When a positive integer n is multiplied by 7, the product is less than 2015. What is the largest possible value of n?

   a) 287 b) 267 c) 277 d) 257 e) 297

2) There were five people in a certain room. Each of them was either a liar who always lies or a knight who always tells the truth. Each of them was asked, "How many liars are among you?" Their answers were "one", "two", "three", "four", and "five", respectively. How many liars were in that room?

   a) 1 b) 2 c) 3 d) 4 e) 5

3) In the five-pointed star below, three angles have the values shown (measured in degrees). What is the measure of $\angle C$?



   a) 35° b) 51° c) 42° d) 109° e) 65°

4) Which of the numbers below divides $10^{2015} + 2$ evenly?

   a) 4 b) 5 c) 6 d) 9 e) 10^{1008}

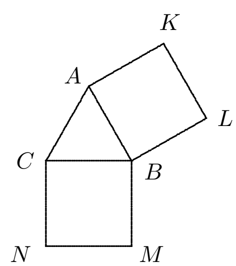
Part B

This part consists of six written-answer questions. For each one, give your solution in the answer book provided. Each question in this part is worth 10 marks. To score full marks, you must provide an answer which is both correct and completely justified.

5) Suppose we toss three dice and add up the number of dots showing. How many different values for the sum can we obtain? (We count the dots showing on the tops of the dice only.)

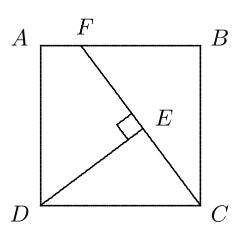
6) The sum of seven consecutive odd numbers equals 119. What is the smallest of these numbers?

7) In the figure below, ABC is an equilateral triangle and $AKLB$ and $BMNC$ are squares. What is the measure of $\angle ALM$?



8) Peter took a quiz with 40 questions. He earned 2 points for each right answer and lost 1 point for each wrong answer. Peter answered all the questions and received a score of 44. How many questions did he answer correctly?

9) In the square $ABCD$ below, DE is perpendicular to FC, $DE = 4$ and $EC = 3$. What is the length of FB?



10) How many pairs of numbers (a, b) satisfy the conditions that $a > b$, $a \times b = 300$, and a and b have a greatest common factor of 1?

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1. When 7 is divided into 2015 the quotient is 287 and the remainder is 6. The largest possible value for n is 287.

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2. It is not possible that two or more persons told the truth. This is because no two people gave the same answer. So, either nobody told the truth or only one person told the truth. Thus there were either four or five liars in the room. If all five persons were lying then the one who said "five" would have told the truth. Since this is impossible, not all five people were lying. The only possibility is that four persons were lying. (One person, the one who said "four", told the truth.)

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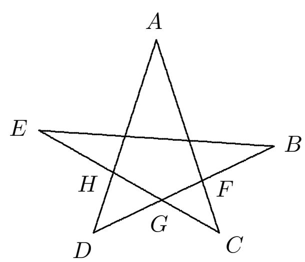
3. Let F, G, and H be the points of intersection shown in the diagram below. It is given that $\angle AHG = 100°$, $\angle HDG = 58°$, and $\angle AFG = 93°$. Since $\angle DHG$ is the supplement of $\angle GHA$, $\angle DHG = 180° - 100° = 80°$. Since the sum of the angles in $\triangle HDG = 180°$, $80° + 58° + \angle DGH = 180°$. It



follows that $\angle DGH = 42°$. By vertical angles, $\angle FGC = 42°$ as well. Also, $\angle GFC$ is the supplement of $\angle GFA$. Thus $\angle GFC = 180° - 93° = 87°$. Finally, since the sum of the angles in $\triangle CFG$ is $180°$, we have $42° + 87° + \angle C = 180°$. Thus $\angle C = 51°$.

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4. Let $n = 10^{2015} + 2$. The numeral for n begins with the digit 1, ends with the digit 2, and has 2014 zeroes between them. It is clear that n is a multiple of 2 and of 3. So, n is a multiple of 6. Also, n is not a multiple of 4. (The last two digits, regarded as the "two-digit" number 02, is not divisible by 4.) Clearly, n is not divisible by 5 or 9, either. Finally, it is obvious that n is not a multiple of 10^{1008}. So, the only number on the list that divides n evenly is 6.

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5. The smallest possible sum is 3. (This happens if each die shows a 1). The largest possible sum is 18. (This happens if each die shows a 6.) Also, any number in between is a possible value for the sum. So, the values for the sum that may be obtained are 3, 4, 5, …, 18. There are 16 such values in all.

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6. Let n be the smallest of the seven odd numbers. The next six odd numbers are $n + 2$, $n + 4$, $n + 6$, $n + 8$, $n + 10$, and $n + 12$. The sum of these numbers is

$$n + (n + 2) + (n + 4) + (n + 6) + (n + 8) + (n + 10) + (n + 12) = 7n + 42.$$

Equating this to 119 gives $7n + 42 = 119$. Dividing throughout by 7 gives $n + 6 = 17$. Subtracting 6 from both sides gives $n = 11$. The smallest of the numbers is 11.

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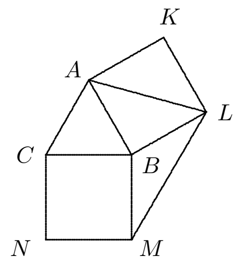
7. Join AL and LM as shown in the figure below. Note that $\angle MBC + \angle CBA + \angle ABL = 90° + 60° + 90° = 240°$. Since the total angle around B is $360°$, $\angle LBM = 120°$. Also, $LB = BA$



(both are sides of the same square), $BA = BC$ (both are sides of the same equilateral triangle), and $BC = BM$ (both are sides of the same square). Thus $LB = BM$. It follows that the triangle BLM is isosceles with $\angle BLM = \angle BML = 30°$. In particular, $\angle BLM = 30°$. Since the diagonal AL bisects the angle KLB, we also have $\angle ALB = 45°$. Then $\angle ALM = \angle ALB + \angle BLM = 45° + 30° = 75°$.

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8. Let x be the number of answers Peter got right, and y the number he got wrong. Since Peter answered every question, $x + y = 40$. Since he received a score of 44, $2x - y = 44$. Adding these equations gives $3x = 84$. Dividing both sides by 3 gives $x = 28$. Peter got 28 answers right (and 12 of them wrong).

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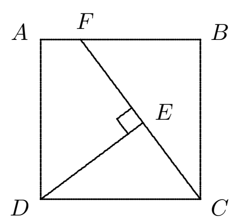
9. The triangle DEC has a right angle at E, and $DE = 4$ and $EC = 3$. By the Pythagorean theorem, $(DC)^2 = (DE)^2 + (EC)^2 = 4^2 + 3^2 = 16 + 9 = 25$. Therefore, $DC = 5$. That is, the side



length of the square $ABCD$ is 5. Also, the triangles DEC and CBF are similar. To see this, note that $\angle DEC = \angle CBF = 90°$, and $\angle DCE = \angle CFB$ (both are complements to $\angle FCB$). It follows that $\angle EDC = \angle BCF$ as well (the sum of the angles in any triangle is $180°$). Since corresponding sides of similar triangles are in the same proportion,

$$\frac{FB}{BC} = \frac{CE}{ED}; \qquad \frac{FB}{5} = \frac{3}{4}.$$

Multiplying the last equation by 5 on both sides gives $FB = 15/4$.

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10. The factors of 300 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 25, 30, 50, 60, 75, 100, 150, and 300. The pairs (a, b) such that $a > b$ and $a \times b = 300$ are:

$$(300, 1), \quad (150, 2), \quad (100, 3), \quad (75, 4), \quad (60, 5), \quad (50, 6), \quad (30, 10), \quad (25, 12), \quad (20, 15)$$

The pairs, $(150, 2)$, $(60, 5)$, $(50, 6)$, $(30, 10)$, and $(20, 15)$ have a common factor greater than 1. (Their greatest common factors are 2, 5, 2, 10, and 5, respectively.) The other pairs, namely $(300, 1)$, $(100, 3)$, $(75, 4)$, and $(25, 12)$, have a greatest common factor of 1. There are 4 such pairs in all.