

The University of the West Indies, Mona Campus
The 2015 Jamaican Mathematical Olympiad

FIRST ROUND EXAMINATION, GRADES 9, 10, AND 11

JANUARY 17, 2015, AT 12:30 PM

Part A

This part consists of four multiple-choice questions. For each one, mark the letter for the correct answer ((a), (b), (c), (d), or (e)) on Page 3 of the answer book provided. Each question in this part is worth 5 marks.

- 1) The result of dividing 20152015 by 2015 is:
a) 11 b) 101 c) 1001 d) 10001 e) 2015

- 2) Let n be equal to the product of all the positive integers from 11 to 29, inclusive. What is the ones digit of n ?
a) 0 b) 2 c) 4 d) 6 e) 8

- 3) How many four-digit numbers with all different digits are divisible by 2014?
a) 1 b) 2 c) 3 d) 4 e) 5

- 4) Each figure below shows a square with a side length of 24 which is filled with one or more shaded circles. In each square, all circles are equal in size and tangent to their neighbors. Circles not surrounded by other circles are tangent to the square they are in. Which figure has the largest amount of shaded area?

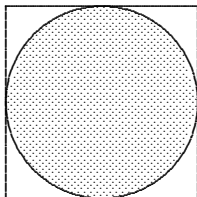


Figure 1

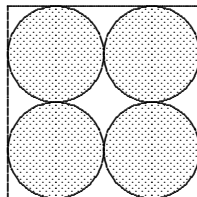


Figure 2

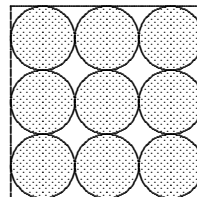


Figure 3

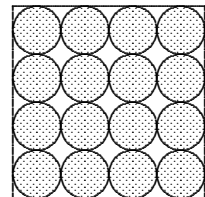


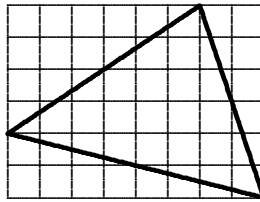
Figure 4

- a) Figure 1 b) Figure 2 c) Figure 3 d) Figure 4
- e) There is an equal amount of shaded area in each figure.

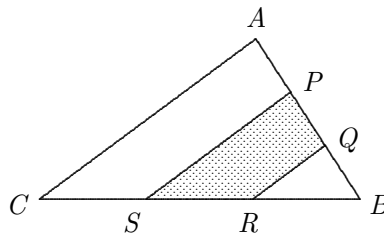
Part B

This part consists of six written-answer questions. For each one, give your solution in the answer book provided. Each question in this part is worth 10 marks. To score full marks, you must provide an answer which is both correct and completely justified.

- 5) Carlos opened his dictionary and said, "If I add the number of the page I am looking for to the number of the next page, I will get 341". What page is Carlos looking for?
- 6) Mario saved \$1,200. One day he spent $\frac{1}{3}$ of his money on a present for his brother. The next day he spent $\frac{1}{4}$ of the remaining money on a book for himself. How much money did he have left then?
- 7) The figure below shows a rectangle divided into squares of size 1×1 . What is the area of the triangle shown in bold?



- 8) The sum of the volumes of three pitchers and two bottles equals 16 litres. The volume of each pitcher is twice as much as the volume of each bottle. What is the total volume of two pitchers and three bottles?
- 9) In triangle ABC below, P and Q divide AB into three equal parts, and S and R divide CB into three equal parts. The area of $\triangle ABC$ is 1. What is the area of the shaded region?



- 10) There were a certain number of people in a room. The average age of these people was equal to the number of people in the room. Then a 29-year-old person came in. It turned out that the average age of the people in the room was once again equal to the number of people in the room. How many people were in the room at the beginning?

The University of the West Indies, Mona Campus
The 2015 Jamaican Mathematical Olympiad

FIRST ROUND SOLUTIONS, GRADES 9, 10, AND 11

JANUARY 17, 2015, AT 12:30 PM

1. When 20152015 is divided by 2015 the quotient is 10001.

2. Note that 20 is one of the numbers between 11 and 29, inclusive. So, the product of the positive integers between 11 and 29, inclusive, is a multiple of 20. Since 20 is a multiple of 10, this product is also a multiple of 10. Its last digit is 0.

3. Suppose n is a four-digit number divisible by 2014. Then n is a four-digit number and is a multiple of 2014. The four-digit multiples of 2014 are 2014, 4028, 6042, and 8056. Each of these numbers has all of its digits different. So, there are 4 such numbers in all.

4. In Figure 1, the radius of the shaded circle is 12. The area of this circle is $\pi(12)^2 = 144\pi$. In Figure 2, the radius of each circle is 6. The area of each circle is $\pi(6)^2 = 36\pi$. The total shaded area

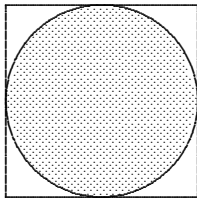


Figure 1

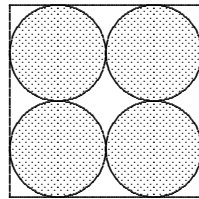


Figure 2

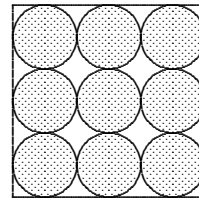


Figure 3

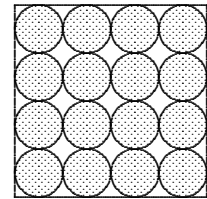


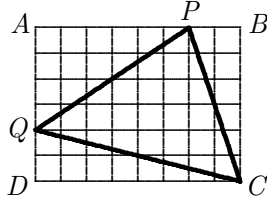
Figure 4

is $4(36\pi) = 144\pi$. In Figure 3, the radius of each circle is 4. The area of each circle is $\pi(4)^2 = 16\pi$. The total shaded area is $9(16\pi) = 144\pi$. In Figure 4, the radius of each circle is 3. The area of each circle is $\pi(3)^2 = 9\pi$. The total shaded area is $16(9\pi) = 144\pi$. So, there is an equal amount of shaded area in each figure.

5. Let p be the number of the page Carlos is looking for. The number of the next page is $p+1$. The sum of these page numbers is $p + (p+1) = 2p+1$. This is equal to 341. That is, $2p+1 = 341$. Subtracting 1 from both sides gives $2p = 340$. Dividing both sides by 2 gives $p = 170$. Carlos is looking for page 170.

6. On the first day, Mario spent $\frac{1}{3} \times \$1200 = \400 . After that, he had $\$1200 - \$400 = \$800$ remaining. On the second day, Mario spent $\frac{1}{4} \times \$800 = \200 on a book. After that, he had $\$800 - \$200 = \$600$ remaining.

7. Let the vertices of the rectangle be A , B , C , and D , and the vertices of the triangle be P , C , and Q



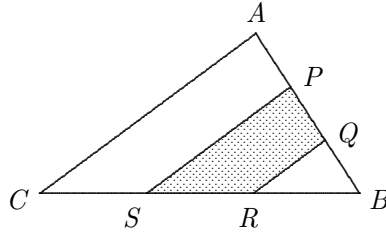
(see the figure above). The area of $ABCD$ is $8 \times 6 = 48$. The area of $\triangle APQ$ is

$$\frac{1}{2}(AP)(AQ) = \frac{1}{2}(6)(4) = 12.$$

Similarly, the area of $\triangle PBC$ is $\frac{1}{2}(2)(6) = 6$ and the area of $\triangle CDQ$ is $\frac{1}{2}(2)(8) = 8$. Then the area of $\triangle PCQ$ is the area of rectangle $ABCD$ minus the combined area of triangles APQ , PBC , and CDQ . This is $48 - (12 + 6 + 8) = 48 - 26 = 22$.

8. Let p be the volume of one pitcher and b the volume of one bottle. (All dimensions in this solution are litres.) It is given that $3p + 2b = 16$, and $p = 2b$. Substituting for p in the first equation gives $3(2b) + 2b = 16$. Thus $6b + 2b = 16$ and hence $8b = 16$. Dividing both sides by 8 gives $b = 2$. One bottle holds 2 litres. It follows that one pitcher holds 4 litres. The total volume, in litres, of 2 pitchers and 3 bottles is given by $2(4) + 3(2) = 8 + 6 = 14$.

9. We have $BQ/BA = BR/BC = 1/3$. Then the triangles BQR and BAC are similar and QR is parallel to AC . Since the scaling factor between $\triangle BQR$ and $\triangle BAC$ is $1/3$, the ratio between



their areas is $1/9$. Since the area of $\triangle BAC$ is 1, the area of $\triangle BQR$ is $1/9$. Similarly, $BP/BA = BS/BC = 2/3$. Then the triangles BPS and BAC are similar and PS is parallel to AC . Since the scaling factor between $\triangle BPS$ and $\triangle BAC$ is $2/3$, the ratio between their areas is $4/9$. Thus the area of $\triangle BPS$ is $4/9$. The area of the shaded region is $4/9 - 1/9 = 3/9 = 1/3$.

10. Let n be the number of people in the room at the beginning, and let a_1, a_2, \dots, a_n be their ages. Since the average age of the people was n ,

$$\frac{a_1 + a_2 + \dots + a_n}{n} = n.$$

Multiplying both sides by n gives

$$a_1 + a_2 + \dots + a_n = n^2. \tag{1}$$

After the 29-year-old person entered, there were $n + 1$ people in the room and their average age was $n + 1$. Thus

$$\frac{a_1 + a_2 + \dots + a_n + 29}{n + 1} = n + 1.$$

Multiplying both sides by $n + 1$ gives $a_1 + a_2 + \dots + a_n + 29 = (n + 1)^2$. Expanding the right sides gives $a_1 + a_2 + \dots + a_n + 29 = n^2 + 2n + 1$. Subtracting equation (1) from both sides gives $29 = 2n + 1$. Solving for n gives $n = 14$. There were 14 people in the room at the beginning