The University of the West Indies

The 2019 Junior Mathematical Olympiad

Solutions For First Round Examination, Grades 5 and 6

Thursday, February 21, 2019

1. We have

$$\frac{2}{1-\frac{2}{3}} = \frac{2}{\left(\frac{1}{3}\right)} = \frac{2}{1} \times \frac{3}{1} = \frac{6}{1} = 6.$$

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2. For the new number to have the highest possible value, the digit 9 should be given the highest possible place value. Then the digit 1 should be changed to 9.

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3. In lowest terms and in improper form,

$$\frac{10}{8} = \frac{5}{4}, \qquad 1\frac{1}{4} = \frac{5}{4}, \qquad 1\frac{3}{12} = 1\frac{1}{4} = \frac{5}{4}, \qquad 1\frac{1}{5} = \frac{6}{5}, \quad \text{and} \quad 1\frac{10}{40} = 1\frac{1}{4} = \frac{5}{4}$$

In particular, $1\frac{1}{5} = \frac{6}{5}$, and this is not equal to $\frac{5}{4}$.

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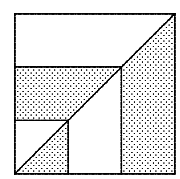
4. Each shaded section is matched exactly by an unshaded section of the same size and shape. One-half of the square is shaded.



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5. The even numbers between 3^2 and 3^3 are the even numbers between 9 and 27. They are 10, 12, 14, 16, 18, 20, 22, 24, and 26. There are 9 such numbers in all.

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6. All together, Maria will be finished 8 hours and 45 minutes after 7:25 am. She will finish at 4:10 pm.

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7. First, $\frac{1}{2}$ of $\frac{2}{3}$ is equal to $\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$. Then $\frac{1}{3}$ of the squares will be removed. That is, $\frac{1}{3}(12) = 4$ squares will be removed. There will be 8 squares remaining.

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8. Since N is between 9 and 17, the average of 6, 10, and N is between the average of 6, 10, and 9 and the average of 6, 10, and 17. The average of 6, 10, and 9 is $8\frac{1}{3}$. The average of 6, 10, and 17 is 11. Among the choices given, the only one between $8\frac{1}{3}$ and 11 is 10.

9. The capacity of the pool is $6\,\text{m} \times 12\,\text{m} \times 4\,\text{m} = 288\,\text{m}^3$. Since the pool is half full, the volume of water in it is $\frac{1}{2}(288\,\text{m}^3) = 144\,\text{m}^3$.

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10. Since $1 + 2 + 3 + 4 + 5 + 6 = 21$, the six weights together weighed 21 kg. The combined weights in the first two boxes weighed $9\,\text{kg} + 8\,\text{kg} = 17\,\text{kg}$. So, the weights in the third box weighed $21\,\text{kg} - 17\,\text{kg} = 4\,\text{kg}$. The weights in the third box had to be the 3 kg and 1 kg weights.

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11. The divisors of 40 are 1, 2, 4, 5, 8, 10, 20, and 40. Among these, 1, 2, 4, and 8 also divide into 72 evenly, but 5, 10, 20, and 40 do not. There are 4 numbers which divide evenly into both 40 and 72.

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12. The sequence 7, 7×7, $7 \times 7 \times 7$, and so on, simplifies to 7, 49, 343, 2401, 16807, 117649, etc. The ones digits form the sequence 7, 9, 3, 1, 7, 9, 3, 1, 7, ... These repeat in cycles of 4. So, the 4th, 8th, 12th, 16th, and so on, up to the 100th term are all 1. If you take the product of 100 sevens the ones digit will be 1

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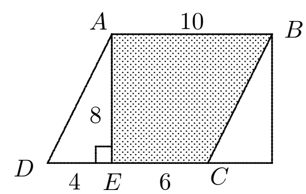
13. Parallelogram $ABCD$ has base width 10 and height 8. Its area is $10 \times 8 = 80$. Since $DC = AB = 10$, we have $DE = 4$. The area of triangle DAE is $\frac{1}{2}(4)(8) = 16$. The area of quadrilateral $ABCE$ is $80 - 16 = 64$.



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a	b	a	11
b	a	c	8
b	c	a	8
10	8	9	

14. From the middle column, it is clear that $a + b + c = 8$. From the top row, we also have $2a + b = 11$. Subtracting the first equation from the second one gives $a - c = 3$. Adding c to both sides gives $a = c + 3$. From the first column, we have $a + 2b = 10$. Subtracting the first equation from this one gives $b - c = 2$. Adding c to both sides gives $b = c + 2$. Now the equation $a + b + c = 8$ becomes $(c + 3) + (c + 2) + c = 8$. This simplifies to $3c + 5 = 8$. Subtracting 5 from both sides gives $3c = 3$. Thus $c = 1$. It follows that $a = 4$ and $b = 3$ as well. Then $a + b - c = 4 + 3 - 1 = 6$.

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15. The only way the sum of the digits of N can be 25 is if they are 9, 9, and 7 (in some order) or 9, 8, and 8 (in some order). In the first case, N could be 997, 979, or 799. In the second case, N could be 988, 898, or 889. There are 6 such numbers in all.