THE 2010 JAMAICAN MATHEMATICAL OLYMPIAD

Presented by The University of the West Indies
In Collaboration with Sterling Asset Management Ltd

Qualifying Round
Test for Grades 9, 10, and 11

NAME: ____________________

GRADE: ____________________

SCHOOL: ____________________

PRINCIPAL: ____________________

DATE OF BIRTH: ____________________

ADDRESS: ____________________

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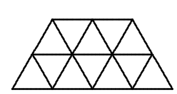
EXAMINATION QUESTIONS

Note: The *natural numbers* are the numbers $\{1, 2, 3, \ldots\}$ These are also called the *counting numbers* or *positive integers*.

1) If four-fifths of a number is 60, what is the number?

(a) 32 (b) 48 (c) 75 (d) 84 (e) 90

2) How many triangles may be found in the diagram below?



(a) 12 (b) 13 (c) 14 (d) 15 (e) 16

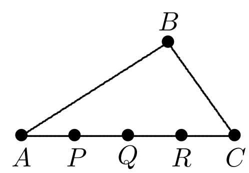
3) Consider the sequence: doh, re, mi, fah, sol, la, ti, doh, re, mi, fah, sol, la, ti, doh, re, mi, ... What is the 2010th term of this sequence?

(a) doh (b) re (c) mi (d) sol (e) ti

4) A certain farmer raises chickens and rabbits. If she has 25 animals and they have a total of 84 legs, how many chickens are on the farm?

(a) 5 (b) 8 (c) 17 (d) 25 (e) 62

5) In the diagram below, ABC is a triangle and P, Q, and R divide AC into 4 equal segments. If the area of $\triangle BQC$ is 24, what is the area of $\triangle BAR$?



(a) 12 (b) 16 (c) 32 (d) 36 (e) 48

6) What is the last digit in the number 3^{2010}?

(a) 1 (b) 3 (c) 5 (d) 7 (e) 9

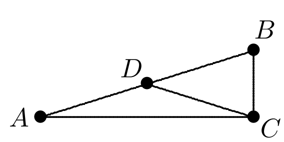
7) Suppose N is a natural number such that $N^2 < 3,456$ and $(N+1)^2 > 3,456$. What is N?

(a) 58 (b) 59 (c) 345 (d) 1728 (e) 3455

8) In a certain town half of the people are men, one-third of the men play dominoes, and one fourth of the male domino players enjoy golf. If 1,200 people live in the town, how many are men who play dominoes and enjoy golf?

(a) 20 (b) 50 (c) 300 (d) 400 (e) 600

9) In the diagram below, ABC is a right triangle and $DB = DC$. If $\angle DAC = 25°$, what is $\angle BDC$?



(a) 25° (b) 40° (c) 45° (d) 50° (e) 60°

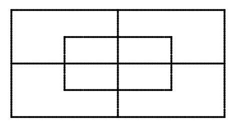
10) How many three-digit numbers are divisible by both 3 and 11?

(a) 25 (b) 26 (c) 27 (d) 33 (e) 105

11) If $a > b > 0$ and $a^2 + 6b^2 = 7ab$, what is a/b?

(a) 1 (b) 6 (c) 7 (d) 13 (e) $\sqrt{42}$

12) How many rectangles may be found in the diagram below?

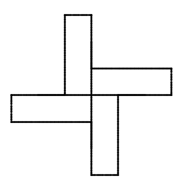


(a) 8 (b) 9 (c) 16 (d) 17 (e) 18

13) What is the sum of the digits in the number $10^{2010} - 2010$?

(a) 9 (b) 2009 (c) 18,007 (d) 18,079 (e) 18,086

14) The figure below consists of four congruent rectangles. The length of each rectangle is three times its width. The area of each rectangle is $12\,\text{cm}^2$. What is the perimeter of the figure?



(a) 6 (b) 12 (c) 16 (d) 48 (e) 64

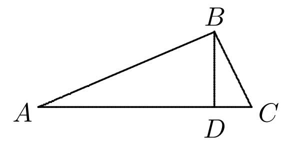
15) How many digits are in the number $(999,999,999,876)^2 - (124)^2$?

(a) 12 (b) 24 (c) 25 (d) 144 (e) 999,876

16) How many natural numbers less than 75 have exactly 4 divisors? (Note: 1 and n are always divisors of n.)

(a) 12 (b) 15 (c) 18 (d) 19 (e) 22

17) Suppose ABC is a triangle with $\angle ABC = 90°$, and let D be the point on AC such that $BD \perp AC$. If $AC = 13$ and $AB = 12$, what is BD?



(a) 4 (b) 56/12 (c) 60/13 (d) 5 (e) 13/2

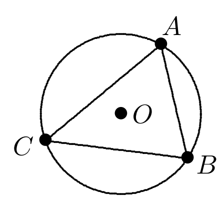
18) In how many ways can we choose natural numbers a and b so that their sum is less than 50 and the equation $x^2 + ax + b$ has exactly one real root?

(a) 0
(b) 3
(c) 5
(d) 6
(e) infinitely many ways

19) Suppose the sequence $a_1, a_2, a_3, a_4, \ldots$ of natural numbers has the property that $a_1 < a_2$ and $a_{n+2} = a_n + a_{n+1}$ for all $n \geq 1$. If $a_6 = 46$, what is a_3?

(a) 2 (b) 8 (c) 10 (d) 18 (e) 23

20) In the diagram below, $\triangle ABC$ is inscribed in a circle with centre O. If $\angle OAB = 40°$ and $\angle OBC = 30°$, what is $\angle OCA$?



(a) 10° (b) 15° (c) 20° (d) 25° (e) 30°

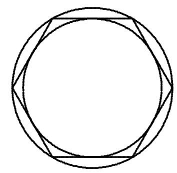
21) If 5^{2010} is divided by 3, what will the remainder be?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

22) A total of 4 packages must be delivered to a building with 5 apartments, called A, B, C, D, and E. If no apartment is receiving more than two packages, in how many ways can they be delivered?

(a) 35 (b) 45 (c) 60 (d) 180 (e) 200

23) A regular hexagon is inscribed in one circle and circumscribed about another. If the inner circle has area A_1 and the outer circle has area A_2, what is A_2/A_1?



(a) $3/4$ (b) $\sqrt{3}/2$ (c) $2/\sqrt{3}$ (d) $4/3$ (e) $\sqrt{3}$

24) Suppose N is a natural number such that $1+2+3+\cdots+N < 1,500$ but $1+2+3+\cdots+N+(N+1) > 1,500$. What is N?

(a) 27 (b) 54 (c) 55 (d) 63 (e) 1,500

25) Suppose S_1 and S_2 are squares with areas A_1 and A_2 and perimeters P_1 and P_2, respectively. If $A_1/A_2 = 5$, what is P_1/P_2?

(a) $1/5$ (b) $\sqrt{5}$ (c) 5 (d) $5\sqrt{5}$ (e) 25

26) In the following multiplication, different letters represent different digits. What is the value of E?

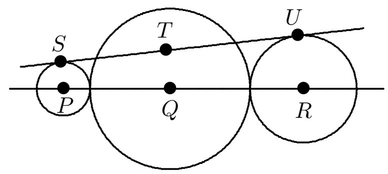
$$\begin{array}{cccc} & & A & B \\ & \times & 5 & D \\ \hline & & A & B \\ B & E & 5 & \\ \hline B & A & 2 & B \end{array}$$

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

27) Suppose a and b are real numbers and $a^2+b^2 = 10$. What is the largest possible value of $2a^2+3b^2$?

(a) 2 (b) 3 (c) 6 (d) 20 (e) 30

28) Circles of radius 2, 6, and 4 are constructed with centres P, Q, and R, respectively. Assume that P, Q, and R lie in a straight line and the circles are tangent. Let SU be the line tangent to the first and third circles. Let T be the point on SU such that $QT \perp SU$. What is the length of QT?



(a) 5/2 (b) 26/9 (c) 3 (d) 11/3 (e) 4

29) How many ordered pairs (m, n) of natural numbers satisfy the relation $m + n < 20$?

(a) 20 (b) 60 (c) 171 (d) 190 (e) 400

30) A six-digit number is called *curious* if its digits are 1, 2, 3, 4, 5, and 6 (in some order) and its first k digits are divisible by k for $k = 1, 2, 3, \ldots, 6$. So, if the number is $d_1d_2d_3d_4d_5d_6$ then d_1 must be divisible by 1, d_1d_2 must be divisible by 2, $d_1d_2d_3$ must be divisible by 3, and so on. How many six-digit numbers are curious?

(a) 0 (b) 2 (c) 3 (d) 6 (e) 16

END OF QUESTIONS

Mail completed question paper to:

Mathematical Olympiad
P.O. Box 94
Mona Post Office
Kingston 7

In order to qualify, all entries must be postmarked no later than February 15, 2010. To be guaranteed consideration, all entries must be received by February 22, 2010.

All high school students in Jamaica are eligible to become a National Mathematics Champion at their grade level. However, only students who were born on January 1, 1994, or thereafter are eligible to be named to the Jamaican Mathematical Olympiad Team.

Please visit http://myspot.mona.uwi.edu/mathematics/ (follow the links below the group picture) for more information and the latest updates.

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