THE 2010 JAMAICAN MATHEMATICAL OLYMPIAD

Presented by The University of the West Indies
In Collaboration with Sterling Asset Management Ltd

Qualifying Round
Test for Grades 7 and 8

NAME: _____

GRADE: _____

SCHOOL: _____

PRINCIPAL: _____

DATE OF BIRTH: _____

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_____

_____

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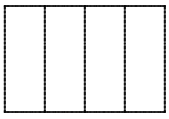
EXAMINATION QUESTIONS

Note: The *natural numbers* are the numbers $\{1, 2, 3, \ldots\}$ These are also called the *counting numbers* or *positive integers*.

1) If 3/4 of a number is 48, what is the number?

(a) 12 (b) 16 (c) 36 (d) 64 (e) 100

2) How many rectangles may be found in the diagram below?

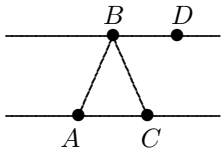


(a) 4 (b) 5 (c) 9 (d) 10 (e) 12

3) The average of three real numbers is 2. If two of the numbers are 5/3 and 7/4, what is the third one?

(a) 6 (b) 9/16 (c) 31/12 (d) 9/5 (e) 2

4) In the diagram below, ABC is an isosceles triangle. Lines AC and BD are parallel. If $\angle ABC = 40°$, what is $\angle CBD$?



(a) 40° (b) 60° (c) 65° (d) 70° (e) 140°

5) In a certain community there are 5 houses. In each house there are 3 children, and each child has 6 toys. How many toys are in the community?

(a) 6 (b) 14 (c) 18 (d) 30 (e) 90

6) What is the next number in the sequence 3, 4, 7, 12, 19, 28, ___, ...?

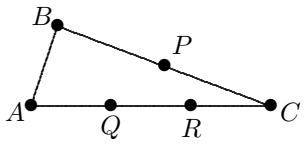
(a) 29 (b) 36 (c) 39 (d) 45 (e) 100

7) In the multiplication below, different letters stand for different digits. What is B?

$$\begin{array}{r} A\ A \\ \times\ A \\ \hline B\ C\ 9 \end{array}$$

(a) 1 (b) 3 (c) 5 (d) 7 (e) 9

8) In the diagram below, ABC is a triangle and P is the midpoint of BC. Also, Q and R divide AC into three equal parts. If the area of $\triangle ABC$ is 36, what is the area of $\triangle PRC$?

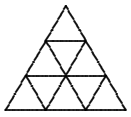


(a) 4 (b) 6 (c) 9 (d) 12 (e) 18

9) What is the last digit in the number 7^{2010}?

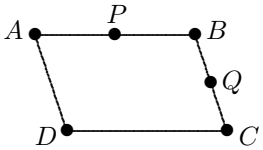
(a) 1 (b) 3 (c) 5 (d) 7 (e) 9

10) A number of straws are used to form a triangular grid in a manner illustrated below. (Only a part of the grid is shown.) If the final grid has 12 straws on each side, how many straws are required to make the grid?



(a) 36 (b) 78 (c) 108 (d) 234 (e) 600

11) In the diagram below, $ABCD$ is a parallelogram. Also, P is the midpoint of AB and Q is the midpoint of BC. If the area of $ABCD$ is 24, what is the area of $\triangle DPQ$?

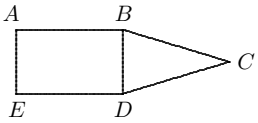


(a) 6 (b) 9 (c) 12 (d) 14 (e) 15

12) A student wishes to write down two distinct letters of the English alphabet in such a way that they are in alphabetical order. In how many ways can this be done?

(a) 25 (b) 26 (c) 210 (d) 325 (e) 351

13) In the diagram below, $ABDE$ is a rectangle. Also, BCD is an isosceles triangle and $AB = BC$. If $\angle BCD = 40°$, what is $\angle AEC$?

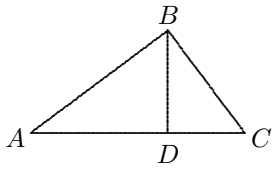


(a) 10° (b) 45° (c) 70° (d) 80° (e) 90°

14) How many positive divisors does 2010 have? (Note: The divisors of a natural number n include 1 and n itself.)

(a) 12 (b) 16 (c) 22 (d) 24 (e) 201

15) Suppose ABC is a triangle with $\angle ABC = 90°$, $AC = 5$, and $BC = 3$. If D is the point on AC such that $BD \perp AC$, what is CD?

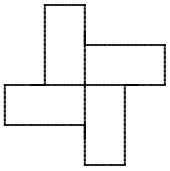


(a) 1 (b) 9/5 (c) 2 (d) 4 (e) 8

16) A two-digit number is called *reflexive* if it is 27 more than the number obtained by reversing its digits. For example, 41 is reflexive because it is 27 more than 14; 81 is not reflexive because it is 63 more than 18. How many two-digit numbers are reflexive?

(a) 1 (b) 3 (c) 5 (d) 7 (e) 9

17) The figure below consists of four rectangles. The length of each rectangle is two times its width and each one has perimeter 18. What is the perimeter of the figure?



(a) 18 (b) 36 (c) 48 (d) 60 (e) 72

18) Suppose a and b are real numbers (not necessarily positive). If $a^2 + b^2 = 4$, what is the largest possible value of $2a^2 - 3b^2$?

(a) 0 (b) 2 (c) 3 (d) 4 (e) 8

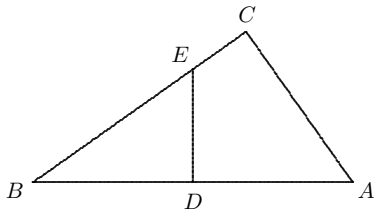
19 If $A + 1 = B - 2 = C + 3 = D - 4 = E + 5$, what is $A + B + C + D + E$?

(a) $5A$ (b) $5A + 1$ (c) $5A + 2$ (d) $5A + 3$ (e) $5A + 4$

20 How many three-digit numbers are odd, divisible by 3, and less than 456?

(a) 58 (b) 59 (c) 60 (d) 116 (e) 117

21) In the figure below, $\angle C = 90°$, $AD = DB$, $DE \perp AB$, $AB = 20$, and $AC = 12$. What is the area of quadrilateral $ADEC$?



(a) 12 (b) 75/2 (c) 117/2 (d) 84 (e) 96

22) In how many ways can 18 be written as a sum of perfect squares?

(a) 1 (b) 8 (c) 9 (d) 10 (e) 12

23) How many natural numbers less than 1,000 are divisible by 4, 5, and 6?

(a) 3 (b) 8 (c) 16 (d) 19 (e) 36

24) Consider the following succession of figures:

How many stars will be in the 8th figure?

(a) 128 (b) 137 (c) 153 (d) 190 (e) 256

25) How many natural numbers have the property that their largest divisor, other than themselves, is 35?

(a) 3 (b) 4 (c) 5 (d) 7 (e) 35

END OF QUESTIONS

Mail completed question paper to:

Mathematical Olympiad
P.O. Box 94
Mona Post Office
Kingston 7

In order to qualify, all entries must be postmarked no later than February 15, 2010. To be guaranteed consideration, all entries must be received by February 22, 2010.

All high school students in Jamaica are eligible to become a National Mathematics Champion at their grade level. However, only students who were born on January 1, 1994, or thereafter are eligible to be named to the Jamaican Mathematical Olympiad Team.

Please visit http://myspot.mona.uwi.edu/mathematics/ (follow the links below the group picture) for more information and the latest updates.

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