2022-2023 Junior Mathematical Olympiad

SOLUTIONS Final Round Examination (Grade 4)

1. **Soln**: **(A)** Since $2 \times 0 = 0$, the value of $2 \times 0 \times 2 \times 3$ is 0.

2. **Soln**: **(C)** Since 60 minutes are in an hour, the number of beats in an hour is $70 \times 60 = 4,200$ times per hour.

3. **Soln**: **(D)** The number of children is the number of girls plus the number of boys. There are 3 girls and 2 boys in this family. The total is 5.

4. **Soln**: **(E)** Of the numbers given, the sum of the hundreds and units digits are $3, 8, 8, 9$ and 3. After tripling them, they are respectively $9, 24, 24, 27, 9$. From this list 9 is the only square and corresponds to the number 231.

5. **Soln**: **(D)** Because Kassie and Zoe were born in the same month, they were both born in March. Because Julie and Zoe were born on the same day of a month, they were both born on the 20th. This means that Helen (the only girl left) was born May 17th.

6. **Soln**: **(D)** $2^2 \times 2^{2020} \times 2 = 2^2 \times 2^{2020} \times 2^1 = 2^{2+2020+1} = 2^{2023}$

7. **Soln**: **(A)** The sum of the 4 sides of the square is $4 \times 10 = 40$ cm. The sum of the 4 sides of the rectangle is $3 + 3 + 10 + 10 = 26$ cm. The difference in length is 40 cm $-$ 26 cm $= 14$ cm

8. **Soln**: **(E)** After putting a melon on the scale with the oranges, the weights of one melon cancel, leaving one melon weighing 6 oranges.

9. **Soln**: **(A)** $\dfrac{66}{77} = \dfrac{6\,(11)}{7\,(11)} = \dfrac{6}{7}$. Similarly the other fractions are $\dfrac{5}{6}$,$\dfrac{4}{5}$ and $\dfrac{3}{4}$. The greatest is $\dfrac{7}{8}$.

10. **Soln**: **(E)** Since Diana is 3 years old and her mother is 28 years older than her, the mother is presently $3 + 28 = 31$ years. In x years, Diana will be $3 + x$ years old and her mother will be $31 + x$ years old. If we solve

$$3\,(3 + x) = 31 + x$$

we get $9 + 3x = 31 + x$ or $2x = 22$ or $x = 11$.

11. **Soln**: **(E)** The number of choices he have is found from the multiplication rule for counting: $2 \times 3 \times 4 = 24$

12. **Soln**: **(D)** Since the remaining parts from using four plates can be used to make one more plate, exactly 3 plates are used to make 4 medals. That is, one medal is made from 3/4 plate of gold. Now $16 = 3 \times 5 + 1$. Fifteen plates will make exactly $5 \times 4 = 20$ medals and the extra plate (the 16th) will make 1 medal. The total is $20 + 1 = 21$.

13. **Soln**: **(E)** Let x be the number of persons that gained MORE than Tony. The number receiving less than Tony is therefore $2x$. The total is therefore $2x + 1 + x = 28$. This gives $x = 9$. This means that $x = 9$ persons got more than Tony and so Tony finished 10th.

14. **Soln**: **(D)** The next number greater than 187569 consisting of different digits is 187590. The difference is $187590 - 187569 = 21$.

15. **Soln**: **(C)** The shaded area is the area of a large rectangle of size $(5 + 6) \times (5 + 3) = 88$ MINUS the area of a smaller rectangle of size $5 \times 6 = 30$. The difference is $88 - 30 = 58$.

16. **Soln**: **(A)** Let today be the day after Jake's birthday. Since the day after tomorrow will be Thursday, tomorrow will be Wednesday which means today is Tuesday and Jake's birthday was on a Monday.

17. **Soln**: **(D)** From the information given 4 pecks = 1 bucket and 9 buckets = 1 barrel. Since $4 \times 9 = 36$ pecks = 9 buckets, 36 pecks = 1 barrel. Peter already picked one peck an so he must pick an additional 35 pecks of peppers.

18. **Soln**: **(C)** In grams, the mass of the largest coin is $100 \times 1000 = 100\,000$. Since a Jamaican \$1 coin has a mass of 10 g. The number of such coins that can be made is $\frac{100000}{10} = 10\,000$ which has a value of \$10 000.

19. **Soln**: **(B)** The greatest three-digit number with all different digits is 987 and the smallest three-digit number with all different digits is 102. The difference is $987 - 102 = 885$.

20. **Soln**: **(C)** If the length of the sides of the squares I, II, III and IV are ℓ_1, ℓ_2, ℓ_3 and ℓ_4 respectively, then $\ell_1 = \frac{16}{4} = 4$ m, , $\ell_2 = \frac{24}{4} = 6$ m. Also $\ell_3 = \ell_1 + \ell_2 = 10$ m and $\ell_4 = \ell_2 + \ell_3 = 6 + 10 = 16$ m. The perimeter is $4\times\ 16 = 64$ m.

21. **Soln**: **(A)** Only when 5 balls are in the bag can we guarantee that she took at least one ball of each color. This would require her to draw at least $14 + 8 + 6 - 5 = 23$ balls.

22. **Soln**: **(C)** The sum of the areas of the three unwanted triangles is

$$\frac{1}{2}(4)(5) + \frac{1}{2}(6)(2) + \frac{1}{2}(2)(1) = 17$$

The area of the required triangle is $24 - 17 = 7$.

23. **Soln**: **(E)** Let c and d be the locations of C and D. because $AB = 2BC$ and $BC = 2CD$,

$$20 - 16 = 2\,(c - 20) \text{ and } c - 20 = 2\,(d - c)$$

The first equation gives $c - 20 = 2$ and $c = 22$. The second equation gives $d - c = 1$ which means $d = 23$.

24. **Soln**: **(C)** If the rays are numbered from 1 (left-most) to 5 (right-most), then the angles are formed form rays

$$(1,2)\,,(1,3)\,,(1,4)\,,(1,5)\,(2,3)\,,(2,4)\,,(2,5)\,,(3,4)\,,(3,5) \text{ and } (4,5)$$

The total is 10. Of this 10, two pairs are equal $(1,3)$ with $(3,4)$ and $(2,4)$ with $(4,5)$. The number with different angle measures is 8.

25. **Soln**: **(D)** Let A be the set of all the multiples of 3 and B the set of numbers ending in 3.

$A = \{3, 6, \ldots, 99\}$ and the number in A is 33. Also, $B = \{3, 13, 23, \ldots, 93\}$ and the number in B is 10. The numbers common to A and B are $3, 33, 63$ and 93 (4 in total). The number of claps is $33 + 10 - 4 = 39$.