UWI's 2022 Junior Mathematics Olympiad

The Date

The University of the West Indies, Mona
presents
The 2022 Junior Mathematics Olympiad

Qualifying Round Examination (SOLUTIONS)

For each question, determine the letter corresponding to the correct or best response; along with the question number, indicate this letter by circling or shading it

1. **Soln**: **(B)** Since 7 is a prime number, the two numbers are 1 and 7 and the sum is 8.

2. **Soln**: **(E)** The cost per marble is $\frac{\$48}{12} = \$4$. Therefore the number of marbles that can be bought with $140 is $\frac{\$140}{\$4} = 35$.

3. **Soln**: **(E)** Using simple ratios, if x is the time in minutes to plant 2500 trees, then
$$\frac{10}{2500} = \frac{3}{x}$$
$10x = 7500$. So $x = 750$. Dividing by 60, the time in minutes is $\frac{750}{60} = 12\frac{1}{2}$.

4. **Soln**: **(C)** Suppose Anna, Buella and Cayla respectively bought a, b, c candies. Then $a = 3b$ and $c = 2a$. Since $c = 18, a = 9$ and $b = 3$.

5. **Soln**: **(B)** The total points after 4 games is $4 \times 31 = 124$. The total points after 5 games is $5 \times 30 = 150$. The difference is the number of points scored in the fifth game. The difference is $150 - 124 = 26$.

6. **Soln**: **(E)** The sum of two odd primes is an even number. Since the sum 85 is odd, one of the primes must be 2, which is the only even prime. The two primes are 2 and 83, so the product is $2 \times 83 = 166$.

7. **Soln**: **(C)** $1000 worth of gas can buy $\frac{1000}{200} = 5$ litres of gas. She can drive 22 km on each gallon, for a total of $22 \times 5 = 110$ km.

8. **Soln**: **(D)** The sum of the areas is $2(1 + 4 + 9 + 16 + 25 + 36) = 182$

9. **Soln**: **(B)** The mother of Mia's daughter is clearly Mia and the mother of the daughter of Mia is again Mia.

10. **Soln**: **(B)** We want the fewest number of digits. Namely $3, 3$ and 4. We compare $502 + 1972 + 970 = 3444$ and $502 + 197 + 2970 = 3669$

11. **Soln**: **(C)** $\frac{b}{a}$ has its maximum value when b is greatest (1200) and a is least (200). In this case, $\frac{b}{a} = \frac{1200}{200} = 6$.

12. **Soln**: **(D)** The octagon can be divided into 8 congruent triangles BOA is one such. 3 complete triangles are shaded and $\frac{1}{2}(\angle BOA)$. The fraction is therefore
$$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{2} \times \frac{1}{8} = \frac{7}{16}$$

13. **Soln**: **(C)** Based on Person 3's response, the number of cars is 7 or the number of houses is 7 but not both. Suppose the number of cars is 7 then the number of houses is neither 7 nor 4. Based on Person 1's response, the number of houses is 6. The total is therefore $7 + 6 = 13$.

14. **Soln**: **(D)** We know that $a + b = 20$ (Bruce's total) and $y + b = 20$ (number of blue marbles). Therefore $y = a$

15. **Soln**: **(C)** Since the heights of the rectangles are all the same, the areas are proportional to the widths:
$$\begin{aligned} 6 &\rightarrow 12 + 18 = 30 \\ CD &\rightarrow 18 + 22 = 40 \end{aligned}$$

The ratios $\frac{6}{CD} = \frac{30}{40} = \frac{3}{4}$. So $3CD = 24$ and $CD = 8$ (cm)

16. **Soln**: **(B)** Using simple ratios, 6 eggs makes $\frac{6}{25} \times 100 = 24$ pancakes, 400g flour makes $\frac{400}{5000} \times 100 = 8$ pancakes, 0.5 liters of milk makes $\frac{0.5}{1} \times 100 = 50$ pancakes and 200g butter makes $\frac{200}{1000} \times 100 = 20$ pancakes. The biggest constraint is on flour.

17. **Soln**: **(C)** If x is the number of days Mary solves problems then Danny solves problems in $(x - 8)$ days. The equal number of problems solved are
$$\begin{aligned} 3x &= 5(x - 8) \\ 3x &= 5x - 40 \end{aligned}$$

Rearranging, $2x = 40$ and $x = 20$. This is the number of days Mary solved problems. Danny solved problems for 12 days. The both solved 60 problems $(20 \times 3 = 60)$

18. **Soln**: **(B)** Because $AB + CD = 137$, we have two choices
$$\begin{aligned} B + D &= 17 \text{ (units) and } A + C = 12 \text{ (tens)} \\ \text{or } B + D &= 7 \text{ (units) and } A + C = 13 \text{ (tens)} \end{aligned}$$

Now, $ADCB + CBAD =$

$$\begin{aligned} & (A+C) \times 1000 + (B+D) \times 100 + (A+C) \times 10 + (B+D) \\ = \; & 13 \times 1000 + 7 \times 100 + 13 \times 10 + 7 \\ = \; & 13\,837 \end{aligned}$$

19. **Soln**: **(E)** The weight of three-fifths jar of water is 740 g -560 g $= 180$ g. Therefore 0ne-fifths jar of water is 60 g. The weight of the jar is therefore 560 g-60 g$=500$ g

20. **Soln**: **(E)** Let the heights be a, b, c, d, e respectively for Andy, Barry, etc. We have

$$\begin{aligned} b & = a+5 \\ b & = c-10 \\ d & = c+10 \\ d & = e-5 \end{aligned}$$

So

$$\begin{aligned} c-10 & = a+5 \\ e-5 & = c+10 \end{aligned}$$

This gives $c = a+15$ and $c = e-15$ or $a+15 = e-15$. That is, $a = e-30$

21. **Soln**: **(D)** Triangle BCD isosceles,so $\angle BCD = \angle CBD = 70°$ and $\angle BDC = 180° - 2 \times 70° = 40°$. Hence $\angle ADB = 180° - 40° = 140°$.

22. **Soln**: **(C)** There are 6 times 4 congruent rectangles contained in the large 12 by 18 rectangle. Only 2 are shaded. The fraction is $\frac{2}{24} = \frac{1}{12}$.

23. **Soln**: **(B)** The first digit can be any of the 9 digits $1, 2, 3, 4, 5, 6, 7, 8, 9$. Given the first digit, there are 9 choices (0 included) for the second digit. There are now 8 and 7 choices for the third and 4th digit respectively. The total number of integers is now $9 \times 9 \times 8 \times 7 = 4536$

24. **Soln**: **(A)** On day 1 the total is 1 on day 2 the total is $1+3 = 4 = 2^2$.On day 3 the total is $1+3+5 = 9 = 3^2$. On day d the total is d^2. On day 20 the total is $20^2 = 400$.

25. **Soln**: **(A)** Because

$$
\begin{aligned}
2022 &= 5\,(404) + 2 \\
404 &= 2\,(202) \\
202 &= 5\,(40) + 2 \\
40 &= 2\,(20)
\end{aligned}
$$

She ended up with 20 $100 bills and change of $22 (2 $1 bills and 2 $10 bill)