2022-2023 Junior Mathematical Olympiad

Round One Solutions (Grades 5 and 6)
1:00pm

For each question, determine the letter corresponding to the correct or best response; along with the question number, indicate this letter by shading it

1. **Soln**: **(A)** Since $2 \times 3 = 6$, the value of $20 + 2 \times 3$ is $20 + 6 = 26$

2. **Soln**: **(B)** To get to the ★, go three units (C) down and three units (3) across, $C3$

3. **Soln**: **(D)** The respective fractions are $\frac{30}{1000}, \frac{48}{1000}, \frac{9}{1000}, \frac{5}{1000}$ and $\frac{20}{1000}$. The smallest is $\frac{5}{1000}$ which is closest to zero.

4. **Soln**: **(E)** It is evident that each row has $3+3 = 6$ seats. The number of rows is therefore $\frac{150}{6} = 25$.

5. **Soln**: **(E)** There are 4 edges at the top, 4 edges at the bottom, and 4 edges at the sides. The total is 12.

6. **Soln**: **(B)** The largest is $54,321$ and the smallest is $12,345$ and the difference is $54,321 - 12,345 = 41,976$.

7. **Soln**: **(E)** Since each boy has least one brother, the number of boys in the family must be at least 2. Since each girl has at least two sisters, the number of girls in the family must be at least 3. The least number of children in the family is $2 + 3 = 5$.

8. **Soln**: **(E)** The largest triangle is made up of 4 smaller triangles and each are made up of 4 smallest triangles (16 in total). $4 + 2 = 6$ are shaded and so the fraction is $6/16 = 3/8$.

9. **Soln**: **(C)** The number of cubes will be the volume of the box in cubic cm. The base has dimensions 4×2 and the height of the box is 2 and so the volume is $4 \times 2 \times 2 = 16$ (cm^3)

10. **Soln**: (**E**) After placing the 1's in their only two possible positions, the completed grid is

1	2		4	3
3	4		2	1
2	1		3	4
4	3		1	2

The total is $1 + 2 + 3 + 4 = 10$

11. **Soln**: (**C**) First, there are $6 \times 8 = 48$ square pieces of chocolate. After eating the outside pieces, the lengths and widths are going to be reduced by 2 (one from top/bottom and left/right). The new dimension will be $4 \times 6 = 24$ and the fraction will be $24/48 = 1/2$

12. **Soln**: (**B**) Since triangle PQR is isosceles, $\angle PQR = \angle PRQ = x°$ and so $70 + x + x = 180$. Since $2x = 110$, $x = 55$. Because $QRST$ is a rectangle, $y = 90$ and $y - x = 90 - 55 = 35$

13. **Soln**: (**A**) Let us undo Harry's moves so we will add 10, half the result and then subtract 5. The result is $\frac{1}{2}(30 + 10) - 5 = 15$.

14. **Soln**: (**C**) The length of a small square is $\frac{20}{5} = 4$ cm. The length of a vertical wire is therefore $3 \times 4 = 12$ cm. Adding the 6 vertical wires and the 4 horizontal wires, we get $6 \times 12 + 4 \times 20 = 152$ cm.

15. **Soln**: (**B**) Since one number is odd and one is even, the smaller of the two numbers is $\frac{1}{2}(317 - 1) = 158$ which is even. Since $158 + 159 = 317$. The odd page number is 159 and the next page number is 160.

16. **Soln**: (**A**) One of each bill will result in $\$5 + \$10 + \$20 + \$50 = \$85$. Since $\frac{510}{85} = 6$, there are 6 sets and the total number of bills is $6 \times 4 = 24$.

17. **Soln**: (**D**) The sentence suggests that $\square 2 \blacksquare = 3\Delta 7 + 76$. Since the sum of the units digit is $7 + 6 = 13$, the value of $\blacksquare$ must be 3. Continuing with the tens place, $1 + 7 + \Delta = 12$ and so $\Delta = 4$. Finally $1 + 3 = \square$ and so $\square = 4$. The sentence is therefore $423 - 76 = 347$ and $\square + \blacksquare + \nabla = 4 + 3 + 4 = 11$.

18. **Soln**: (**C**) The number of grams in the special packet is $\frac{4}{3} \times 75$ g$= 100$ g.

19. **Soln**: (**C**) Case 1 (Andrew is a liar): Suppose Andrew is a liar then Bill is truthful which means Clair is a liar which means Daniel is truthful and Eva would be a liar. This result in 3 liars.

Case 2 (Andrew is truthful): Suppose Andrew is truthful then Bill is a liar which means Clair is a truthful which means Daniel is a liar and Eva would be truthful. This result in 2 liars.

20. **Soln**: (**E**) There are triangles of 5 different sizes. Counting triangles, largest to smallest, the total is

$$1 + 6 + 3 + 1 + 6 = 17$$

21. **Soln**: (**D**) The bus cover 4 "travels" and 3 "waits". The total travel and wait time is $4 \times 10 + 3 \times 3 = 49$ minutes. Now,

$$10 : 35 \text{ am} + 49 \text{ mins } = 10 : 84 \text{ am} = 11 : 24 \text{ am}$$

22. **Soln**: (**E**) Since 2/3 cup of milk makes 12 muffins, 2 cups of milk will make $3 \times 12 = 36$ muffins. So one cup will make 18 muffins and 18 cups of milk will make $18 \times 18 = 324$ muffins.

23. **Soln**: (**C**) Any one of the triangle may be divided into $3 + 6 = 9$ congruent triangles, six of which are shaded to form the hexagon. The area of the hexagon is therefore $\frac{6}{9} \times 36$ cm^2 = 24 cm^2.

24. **Soln**: (**E**) The number takes the form $4n + 1, 5m + 3, 6r + 1$ for integers n, m and r. The numbers in the set satisfying $6r + 1$ are $43, 49, 55, 61, 67, 73, 79$. Of this set, the numbers satisfying $5m + 3$ are 43 and 73 and from this set only 73 satisfies $4n + 1$.

25. **Soln**: (**A**) Let the length one side of the square be x.

$$x + a = 6 \text{ and } x + b = 8$$

where the length of the rectangle is $b + x + a$ and the width is x. The half-perimeter is

$$b + x + a + x = b + 2x + a = (x + a) + (x + b) = 4 + 8 = 14$$

The perimeter is therefore 14 cm + 14 cm = 28 cm.