COURSE NAME AND CODE: Introduction to Formal Mathematics (MATH1152)

LEVEL: I

NUMBER OF CREDITS: 3

PREREQUISITES: CAPE or GCE A-Level Mathematics, or M08B/MATH0100 and M08C/MATH0110, or equivalent

RATIONALE: Every branch of higher mathematics is defined by its axioms and characterized by the theorems one may deduce from them. This understanding of mathematics is in contrast to the mathematics that first-year students will consider familiar, which emphasizes computational processes guided by intuitive understanding. This course is designed to both enrich students' mathematical foundations and teach them the elements of proving theorems. This will prepare them for success with the higher mathematics they will normally encounter as they proceed through a modern mathematics major.

COURSE DESCRIPTION: This is a first-year course required for all students who will major in mathematics. It presents the axioms of the familiar number systems (the natural numbers, the integers, and the reals) and deduces basic facts concerning each system. To facilitate this process, basic methods of logic, deduction, and proof are introduced at the outset and repeatedly applied throughout the course.

LEARNING OUTCOMES: At the end of the course, students will be able to:

1. test simple logical equivalences (for example, $\neg(p \Rightarrow q) \equiv p \wedge \neg q$), and various patterns of reasoning for validity;

2. demonstrate an ability to use case-by-case reasoning, construct proofs by contradiction, and proofs by induction (once relevant material has been introduced);

3. test binary operations to determine whether they are associative, commutative, etc;

4. test relations to determine whether they are equivalence relations and, if so, produce equivalence classes as appropriate;

5. prove elementary facts about the natural numbers directly from the axioms and relevant definitions;

6. prove elementary facts about the integers directly from the axioms and relevant definitions;

7. prove facts about divisibility such as:
    a) If a divides b and b divides c then a divides c;
    b) If a divides b and a divides c then a divides $b \pm c$.

8. Apply the Euclidean algorithm to produce the GCD of two (nonzero) integers.

9. Prove elementary facts about the real numbers directly from the definitions, axioms, and elementary theorems already established. For example students should be able to prove that:
    a) If a, b, c, d are real numbers then $[(a + b) + (c + d)] = d + [a + (c + b)]$
    b) If a, b, c, d are positive real numbers with $a < b$ and $c < d$ then $ac < bd$;
    c) If a and b are positive numbers and $a < b$ then $a^n < b^n$ and $\sqrt[n]{a} < \sqrt[n]{b}$ for all natural numbers n.
    d) Prove that numbers such as $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \cdots$ are irrational.

CONTENT:

1. Formal Symbolic Logic (6 hours).
    [Lectures 1–3] Statements, negation, and the connectives $\wedge$, $\vee$, and $\Rightarrow$ with examples
    [Lecture 4] Truth tables, testing reasoning patterns with truth tables
    [Lecture 5] Case-by-case analysis, with examples
    [Lecture 6] Proof by contradiction, with examples

2. Sets, Relations, and Equivalence Relations (5 hours).
    [Lecture 7] Basic set theory
    [Lectures 8–9] Relations and their properties: reflexive, symmetric, and transitive
    [Lectures 10–11] Equivalence relations; equivalence classes.

3. Binary Operations (5 hours).
    [Lecture 12] Operations as a mapping from $A \times A \to A$;
    [Lectures 13–14] Associativity and commutativity
    [Lectures 15–16] Identity elements and inverses

4. The Natural Numbers (5 hours).
    [Lecture 17] The axioms of the natural numbers
    [Lecture 18] Addition of the natural numbers
    [Lecture 19] Multiplication of natural numbers
    [Lecture 20] Elementary proofs (e.g., if $ab = ac$ then $b = c$)
    [Lecture 21] The Principle of Mathematical Induction.

5. The Integers (9 hours).
    [Lecture 22] The axioms of the integers
    [Lectures 23–24] Elementary proofs
    [Lectures 25–26] Divisibility and basic proofs
    [Lecture 27] The unique prime factorization of an integer
    [Lectures 28–29] The Euclidean algorithm
    [Lecture 30] Remainder classes.

6. The Real Numbers (9 hours).
   [Lectures 31–33] The axioms of addition and elementary proofs
   [Lectures 34] The axioms of multiplication and elementary proofs
   [Lectures 35] The distributive law with elementary proofs
   [Lectures 36–37] The axioms of order with elementary proofs
   [Lectures 38–39] The axiom of completeness with proofs

TEACHING METHODOLOGY This course will be delivered by a combination of lectures and tutorials. The total of 39 contact hours is broken down as follows: 28 hours of lectures and 11 hours of tutorials. The tutorial will be interspersed with the lectures by having students discuss exercises, revise material as needed, and cover new content each day. Course materials such as exercises, assignments, solutions, etc., will be posted on the webpage http://ourvle.mona.uwi.edu/

ASSESSMENT. The course assessment will be broken into two components; a course- work component worth 40% and a final exam worth 60%. The Coursework element will consist of two distinct parts:

i. The Final exam will be two hours in length.
ii. One course work exams that should take place during week 9. This exam will be worth 25% of the student's final grade.
iii. Five take home assignments. One will be given every fortnight. These assignments will count for 15% (with each one being worth of 3%) of the student's final grade.

COURSE TEXTS:

1. Gary Chartrand, Albert D. Polimeni, and Ping Zhang, Mathematical Proofs, A Transition to Advanced Mathematics, Pearson International Edition, 2008
2. W. Edwin Clark, Elementary Abstract Algebra. This may be downloaded for free at http:// shell.cas.usf.edu/ wcalrk