COURSE NAME:	ELEMENTS OF MATHEMATICAL ANALYSIS
COURSE CODE:	MATH 2401
LEVEL:	II
SEMESTER:	Ι
NUMBER OF CREDITS:	3
PREREQUISITES:	(MATH1141, MATH1142, MATH1151 and MATH1152) or (M10A, M10B)

### **RATIONALE:**

This course aims to give students a substantial knowledge of the concepts: limits, continuity, differentiation, experience with epsilon-delta definitions and the construction of proofs. It is a compulsory course for student taking the Mathematics major and could be useful for students taking the Physics major. It is a required foundation for the more advanced mathematics courses.

## **COURSE DESCRIPTION:**

This is a classical course in analysis, providing a foundation for many other mathematical courses. The course exposes students to rigorous mathematical definitions of limits of sequences of numbers and functions, classical results about continuity and differentiability, series of numbers and functions, and their proofs. A particular focus of the course is in providing students with practical expertise working with rigorous definitions and creating proofs. The following topics will be covered: sequences, continuity, differentiability, series of numbers, and series of functions.

# **LEARNING OUTCOMES:**

By the end of the course, students will be able to:

- prove that a given number is a limit of given sequence or function;
- prove several classical theorems from analyses;
- prove that a given function is continuous or discontinuous and classify the points of discontinuity;
- prove that a given function is a derivative of another function;
- justify the convergence/divergence of a given number series;
- derive numerical approximations via Taylor series.

# CONTENT:

- Sequences:
  - The least upper and the greatest lower bounds; the Completeness axiom, sequences, limits; bounded, monotone and Cauchy sequences; Convergence theorem; subsequence; the Bolzano-Weierstrass theorem; limsup, liminf.
- Limits and Continuity:

- The limit of functions, left and right limits, properties; lim sin x/x, and lim(1+x)^x; continuity, different types of discontinuity; properties of continuous functions on close interval; intermediate and extreme values; uniform continuity
- Differentiability:
  - Derivative; the Mean-Value theorem; inverse function.
- Infinite Series:
  - Convergence of infinite series; the divergence test, positive series tests (comparison, limit comparison, ratio, root); absolute convergence; alternating series; Cauchy criterion for convergence.
- Sequence and Series of functions:
  - The pointwise convergence of a sequences of functions; uniform convergence of sequences of functions; uniform convergence of series of functions; convergence of power series; Abel's and Weierstrass's tests; functions defined by power series; Taylor series.

### **TEACHING METHODOLOGY:**

Students will be exposed to the theoretical aspects of the mathematical analysis through informative lectures. Tutorials provided as needed during scheduled lecture hours will reinforce their learning and provide experience with the practical application of different type of problems.

The total estimated 39 contact hours may be accounted for as follows: 26 hours of lectures and 13 hours of tutorials. Course material, including practice problems, will be posted on the webpage: <u>http://ourvle.mona.uwi.edu/</u>

**ASSESSMENT:** The course assessment has three components:

Final exam: 2-hour written paper	70%
Two Midterm Exams (10% each)	20%
Two Written Assignments (5% each)	10%.

## **REFERENCE MATERIAL:**

#### **Books:**

Prescribed

LANG, Serge. *Undergraduate Analysis* (Undergraduate Texts in Mathematics), second edition, Springer, 2010. ISBN: 0387948414.

#### Recommended

ABBOTT, Stephen. *Understanding Analysis* (Undergraduate Texts in Mathematics), Springer, 2010. ISBN: 9781441928665.

Highly Recommended

LANG, Serge. *Undergraduate Analysis* (Undergraduate Texts in Mathematics), second edition, Springer, 2010. ISBN: 0387948414.

These books are pedagogically sound, comprehensively address all element of the syllabus, and provide useful case studies and examples.

# **Online Resources:**

# 1. <u>http://mathforum.org/library/topics/real\_a/</u>

# The Math Forum Internet Mathematics Library.

This is a list of online resources for Real Analysis, including online lecture notes, software. The site is maintained by the Goodwin College of Professional Studies at Drexel University.

2. http://www.jirka.org/ra/

This free on-line textbook is a one semester course in real analysis, taught in the fall semester of 2009 at the University of Illinois at Urbana –Champaign.