COURSE NAME:	**STOCHASTIC MODELING**
COURSE CODE:	MATH2407
LEVEL:	II
SEMESTER:	II
NUMBER OF CREDITS:	3
PREREQUISITES:	MATH2404

RATIONALE

Randomness is highly influential in real-world systems and there is a large body of theory available allowing random systems to be modeled and analyzed. It is important to provide students with a theoretical knowledge of this field, while at the same time developing in them the skills and experience required to develop high quality models in practice. This will be of great value to students preparing for careers in financial services, engineering, and the design and management of complex systems. The course will also provide a foundation in the theory of stochastic processes necessary for further study in mathematical finance.

COURSE DESCRIPTION:

The course introduces several of the main classes of stochastic process. Their properties are studied, and various case study examples are used to show how they may be used in the construction of real-world models. The course also provides students with practical experience of simulation, and, as part of a group project, requires them to construct and analyse a stochastic model using modern mathematical software.

LEARNING OUTCOMES:

By the end of the course, students will be able to:

- classify a stochastic process according to the nature of its time set and state space;
- illustrate the Strong Law of Large Numbers and the Central Limit Theorem via simulation;
- construct a Markov chain model, and analyse its properties via the transition matrix;
- identify Poisson process, and derive their basic properties;
- classify queuing processes, and use them in modeling;
- analyse the Gambler’s Ruin problem via random walks and Brownian motion;
- construct and analyse in detail a stochastic model using mathematical software, as part of a group project.

CONTENT:

Introduction:

- Significant discrete and continuous random variables and their probability distributions;
- Sums of random variables: convolution and their distribution;
- Conditional probability and conditional expectation;
- Introduction to stochastic processes: definition, time set & state space classifications.

Markov processes:

- Time homogeneous and inhomogeneous Markov chain: one-step transition probabilities, one-step transition matrix, kth-step transition probabilities, limiting distributions;
- Random walk: absorbing states, first passage times, mean time to absorption, recurrence, Gambler's Ruin problem;
- The homogeneous Poisson process: exponential successive inter-arrival times; waiting times, sojourn times, transition times;

Queues:

- The Bernoulli single server queuing process: limited and unlimited capacity queues, arrival process, service process;
- M/M/1 queuing process, limiting distributions;
- M/M/k queuing process;

Brownian motion:

- Motivation and definition;
- Properties: the reflection principle, first hitting times, zeros of Brownian motion;
- Brownian motion with drift.

Laboratory work:

- Probability basics, random variables and distributions;
- Pseudo-random number generators;
- Markov chains, Poisson processes, queues and Brownian motion: applications and simulation;
- Supervised group project work.

TEACHING METHODOLOGY:

Students will be exposed to the theoretical aspects of stochastic processes and modeling through informative lectures. Tutorials (provided as needed during scheduled lecture hours) will reinforce their learning and provide experience with the practical application of stochastic models. In parallel with this students will attend computer laboratory sessions where they will implement and explore the ideas and models encountered in the course.

The total estimated 39 contact hours may be accounted for as follows: 32 hours of lectures and 14 hours of supervised laboratory time (counted overall as 7 contact hours). Course material, including practice problems, will be posted on the webpage

http://ourvle.mona.uwi.edu/

ASSESSMENT:

The course assessment has three components:

1. One in-course test [1 hour] - 20%
2. One Group project - 20%
3. Final theory exam [2 hours] - 60%

- The group project will be laboratory based and require the analysis of one or more aspects of stochastic modeling in software. The size of each group will be 2-3 students, and a common mark awarded to each student in a given group based on the group submission.

REFERENCE MATERIAL:

Books

Prescribed

1. Sheldon M. Ross, *Introduction to Probability Models (Ninth Edition)*, Academic Press, 2006. ISBN-10: 0125980620

Highly Recommended

2. Howard Taylor, Samuel Karlin, *An Introduction to Stochastic Modeling (Third Edition)*, Academic Press, 1998. ISBN-10: 0126848874

These books are pedagogically sound, comprehensively address all element of the syllabus, and provide useful case studies and examples.

Online Resources:

1. http://www.random.org/ - This website presents a well-developed discussion of the concept of randomness, and the distinction between pseudo- and true-random numbers. It allows the students to generate large quantities of true-random numbers (generated from atmospheric noise) for use in simulations, which will complement their work with pseudo-random generators in laboratory sessions.
2. http://mathforum.org/library/topics/probability/ - The MathForum Internet Mathematics Library is a curated list of online resources for Probability Theory, including online lecture notes, software, and pages addressing famous problems like Monty Hall and the Prosecutor's Fallacy. The site is maintained by the Goodwin College of Professional Studies at Drexel University.