

<b>COURSE NAME:</b>	Ordinary Differential Equations
<b>COURSE CODE:</b>	MATH 2420
<b>LEVEL:</b>	II
<b>SEMESTER:</b>	I
<b>NUMBER OF CREDITS:</b>	3
<b>PREREQUISITES:</b>	(MATH 1141, MATH 1142, MATH 1151 & MATH 1151) or (M10A & M10B)

### **RATIONALE:**

MATH2300 is the first course in a two-semester sequence in differential equations. The goal of this sequence is to provide students a solid foundation in differential equations so that they will be adequately prepared for subsequent courses in mathematics, physics, and other scientific disciplines. The topic of differential equations is an extremely important one in mathematics and science, as well as many other branches of studies (economics, chemistry, commerce) in which changes occur and in which predictions are desirable. In most such circumstances, the systems studied come with some kind of "Natural Laws", or observations that, when translated into the language of mathematics, become differential equations. It is then the job of the scientist to try to figure out what are the predictions, i.e. to find the functions that satisfy those equations.

### **COURSE DESCRIPTION:**

This is a Level II compulsory course for majors and minors in Mathematics, which is suitable for all science students. This course introduces methods for solving first- and second-order ordinary differential equations, systems of differential equations, and power series solutions. The purpose of this course is to introduce students to the theory and application of differential equations. Specifically, the course prepares students to apply differential equations to scientific, engineering and economic problems. Furthermore, students will be exposed to modern mathematical software (Math Lab or Maple) to explore the concepts encountered in the course.

### **CONTENT:**

**Classification of Differential Equations – First Order Differential Equations – Second Order linear Differential Equations – Power Series Solutions**

### **LEARNING OBJECTIVES:**

At the end of the course, students will be able to:

- Classify differential equations into linear/nonlinear, exact/non-exact, separable, autonomous and non-homogeneous second-order differential equations;
- Solve separable first-order differential equations;
- Solve exact first-order differential equations;
- Solve certain classes of non-exact first-order differential equations;
- Analyze the domain of validity of solutions of first-order differential equations;
- Solve homogeneous second-order differential equations with constant coefficients;
- Solve non-homogeneous second-order differential equations with constant coefficients;
- Solve certain classes of variable coefficient second-order linear differential equations;
- Classify the singularity of second-order linear differential equation with variable coefficients;
- Solve second-order linear equations using power series methods and the method of Frobenius.

## **CONTENT**

### **Classification of Differential Equations:**

Ordinary and partial differential equations, systems of differential equations, order of a differential equation, linear and nonlinear equations, what is a solution of a differential equation.

### **First Order Differential Equations:**

Linear equations with variable coefficients, separable equations, test of exactness, non-exact differential equations and integrating factors, the existence and uniqueness theorems for first-order linear and nonlinear differential equations (without proofs), interval of definition, differences between linear and nonlinear equations, Picard's method of successive approximations.

### **Higher Order Linear Equations:**

Homogeneous equations with constant coefficients, fundamental solutions of linear homogeneous equations, linear independence and the Wronskian, complex roots of the characteristic equation, repeated roots, reduction of order, nonhomogeneous equations and general formula for the solution involving the Wronskian.

### **Power series solutions:**

Short review of power series and convergence tests, Taylor series and analytic functions, standard form of second order linear differential equations, ordinary and singular points, power series solution of second order linear differential equations around a regular point,

recurrence relation, gymnastics in shifting the index of summation; regular and irregular singular points, method of Frobenius, the indicial equation and the exponents at the singularity.

### **Legendre polynomials and Bessel functions:**

Fuchs theorem, general considerations on the convergence radius of series solutions for the Legendre and Bessel equations around an ordinary point, elementary and special functions, the Legendre equation: solutions around  $x=0$ , Legendre polynomials; Bessel equation of order  $\nu$ , Bessel functions of fractional order, Bessel function of order zero of the first kind, Bessel function of order  $\nu$  of the first kind and its asymptotic behavior for large  $x$ , Gamma function and Bessel function of arbitrary order.

Tutorials

## **TEACHING METHODOLOGY**

This course will be delivered by a combination of interactive lectures and participative tutorials. The total estimated 39 contact hours are broken down as follows: 26 hours of lectures, 13 hours of tutorials. The course material will be posted on the webpage

<http://ourvle.mona.uwi.edu/>

Practice problems and assignments will also be available to students via this webpage, as well as the solutions to the assignment questions after the due date.

## **ASSESSMENT**

The course assessment will be divided into two components: a coursework component worth a 30% and a final exam worth a 70%.

- Two one-hour course work exams will take place during weeks 5 and 9. Each will be worth a 10% of the student's final grade.
- Written assignments (problem papers), total grade worth a 10% of the student's final grade.
- The final theory exam will be two hours in length and consist of compulsory questions.

## **REFERENCE MATERIAL:**

### **Books:**

1. BOYCE, W. E. & R. C. DiPrima. *Elementary Differential Equations and Boundary Value Problems*. John Wiley & Sons, 8<sup>th</sup> Edition, 2005. ISBN 0-471-43338-1 [prescribed]
2. AGARWAL, Ravi P. & Doanl O'Regan. *An Introduction to Ordinary Differential Equations*. Springer, 2008. ISBN-10: 9780387712758 [highly recommended]

3. BIRKHOFF, G. & G. Rota: *Ordinary Differential Equations*, John Wiley & Sons, 4<sup>th</sup> Edition, 1989. ISBN: 0-471-86003-4 [recommended]

**Online Resources:**

<http://mathforum.org/differential/differential.html> - Links to some of the best Internet resources for differential equations: classroom materials, software, Internet projects, and public forums for discussion

<http://www.sosmath.com/diffeq/diffeq.html> - An online course: learning units presented in worksheet format review the most important results, techniques and formulas in college and pre-college differential equations. Sections include: Introduction and First Definitions; Modelling via Differential Equations; First, Second, and Higher Order Differential Equations; Laplace Transform; Systems of Differential Equations; Fourier Series; and an Appendix with Mathematical Tables and Formulas.

<http://tutorial.math.lamar.edu/classes/de/de.aspx> - online notes for Ordinary Differential Equations taught at Lamar University. They are accessible to anyone wanting to learn Differential Equations or needing a refresher in some of the topics from the class.