COURSE NAME: Complex Variables

COURSE CODE: MATH3155

LEVEL: III

SEMESTER: I

NUMBER OF CREDITS: 3

PREREQUISITES: MATH2401

RATIONALE:

Complex analysis is a particularly beautiful area of mathematics which may be studied in its own right. However, its significance extends further: functions of complex variables arise in many other branches of mathematics both as part of a theoretical framework (e.g. the Riemann zeta function in number theory) and in applied areas like electronics, time series analysis and physics. Providing students with a background in this field is an essential part of developing mathematical maturity, and prepares them for advanced work in many other areas.

COURSE DESCRIPTION:

This course develops the properties of the complex number system, treated as a generalization of the real number system. We explore the parallel analysis that results, with a particular emphasis on the development of integral calculus in the complex realm.

LEARNING OUTCOMES:

By the end of the course, students will be able to:

- Identify and construct analytic functions using the Cauchy Riemann equations;
- Use branch cuts to construct an analytic function from a multi-valued elementary function;
- Compute contour integrals via path parameterization;
- Apply the Cauchy-Goursat theorem, the Cauchy integral formula and the principle of deformation of path to compute contour integrals;
- Develop Taylor and Laurent series representations for elementary functions;
- Classify isolated singular points and compute residues;
- Use the Residue Theorem to evaluate improper real integrals.

CONTENT:

Review of complex numbers:

- Algebraic and geometric representation of complex numbers;

- Euler's formula;
- Rational powers and roots of complex numbers;
- Regions in the complex plane.

Analytic functions:

- Limits, continuity and differentiability;
- Cauchy Riemann equations;
- Analytic and harmonic functions;

Elementary functions:

- The complex exponential function;
- Trigonometric and Hyperbolic functions and inverses;
- The complex logarithm – definition, properties, branches and branch cuts;
- Complex powers.

Integrals:

- The contour integral – definition, properties, application;
- Bounds on integrals;
- Antiderivatives;
- The Cauchy-Goursat theorem and the principal of deformation of path, Cauchy's integral formula;
- Cauchy's inequality and the Maximum Modulus Principle;

Series:

- Convergence of sequences and series;
- Power series – absolute and uniform convergence, integration and differentiation;
- Taylor and Laurent series;

Residues and Poles

- Isolated singular points, residues and the Residue Theorem;
- Classifying isolated singular points;
- Residues at poles;
- Evaluation of improper real integrals by contour integration around poles.

TEACHING METHODOLOGY:

The course is theoretical in nature and will be deliver via informative lectures supported by tutorials (provided as needed during scheduled lecture hours) designed to give students practical self-guided exposure to the ideas presented in lectures.

The total estimated 39 contact hours may be accounted for as follows: 26 hours of lectures and 13 hours of tutorial time. Course material, including assignments and practice problems, will be posted on the webpage

http://ourvle.mona.uwi.edu/

ASSESSMENT:

The course assessment has three components:

1. One in-course test - 20% of overall grade;
2. Two assignments – 10% each totalling 20% of overall grade;
3. Final exam - 60% of overall grade.

The final exam will be two hours in length and consist of compulsory questions.

REFERENCE MATERIAL:

Books:

Prescribed: James Ward Brown, Ruel V. Churchill. *Complex Variables and Applications (Eighth Edition).* McGraw-Hill College, 2008.

This book is pedagogically sound, comprehensively addresses all element of the syllabus, and provides useful case studies and examples.

Online Resources:

1. http://mathforum.org/library/topics/complex_a/ - The MathForum Internet Mathematics Library is a curated list of online resources for Complex Analysis, including online lecture notes, software, and practice problems. The site is maintained by the Goodwin College of Professional Studies at Drexel University.
2. http://www.math.ttu.edu/~pearce/complex.shtml - This page contains a list of websites that allow complex mappings to be visualized.