COURSE NAME: Introduction to the Theory of Integration

COURSE CODE: MATH 3401

LEVEL: III

NUMBER OF CREDITS: 3

SEMESTER: I

PREREQUISITES: MATH2401

RATIONALE: The idea of integration is fundamental in mathematics. This course observes the inherent limitations of the Riemann integral, which may be overcome by developing a mathematical notion of 'length'. Following this, the more general Lebesgue integral may be developed, and its special properties examined. Providing students with a background in this field is an essential part of developing mathematical maturity, and prepares them for advanced work in many other areas.

COURSE DESCRIPTION:

This is a Level III course for majors and minors in Mathematics, which is suitable for mathematics and physics students. This course shows that the Riemann integral fails for certain classes of functions, introduces the mathematical concept of measure, and uses it to develop the Lebesgue integral, culminating in a treatment of the Monotone and Dominated Convergence Theorems.

LEARNING OUTCOMES:

At the end of the course, students will be able to:

- Explain the structure of the Riemann integral and identify where it breaks down;
- Demonstrate that the Dirichlet function is not Riemann integrable;
- Explain the concept of measure in the line;
- Formulate main properties of the Lebesgue integral;
- Prove Monotone and Dominated Convergence Theorems;
- Present examples demonstrating difference between the Riemann and the Lebesque integrals.

CONTENT:

REIMANN INTEGRAL: Definition and existence of the definite integral. Darboux sums. Upper and low sums. Mean Value theorems. Reimann integral as a function of the upper limit. The Dirichlet function.

MEASURABLE SETS ON A LINE: Open and Closed Sets, Intuitive meaning of Lebesgue measure; Sets of Measure Zero; Compact Sets, Heine-Borel Theorem.

LEBESGUE INTEGRAL: Step functions on an Interval, the integral of the step function; properties; upper functions on the interval; Lebesgue integrable functions on the interval; properties, Lebesgue integral on a set of measure zero; connection with Riemann integration; integral of the Dirichlet function.

MONOTONE AND DOMINATED CONVERGENCE THEOREMS: Monotone convergence theorem for step functions, for upper functions and for Lebesgue integrable fuctions on the interval, Lebesgue's Theorem, consequences of Lebesgue's Theorem.

TEACHING METHODOLOGY

The abstract concepts, illustrated with examples, will be presented during the lectures. The total estimated 39 contact hours may be accounted for as follows: 26 hours of lectures and 13 hours of tutorials. Course material, including practice problems, will be posted on the webpage http://ourvle.mona.uwi.edu/.

ASSESSMENT:

The course assessment has three components:

- One in-course test (1 hour) - 20% of overall grade.
- Two assignments - 10% each totalling 20% of overall grade.
- Final examination - 60% of overall grade.

The final examination will be two hours in length and consists of compulsory questions.

REFERENCE MATERIAL:

Books:

Prescribed:

1. H.S. Bear, A Primer of Lebesgue Integration (Second Edition). Academic Press, 2002. ISBN-10: 0120839717

Highly recommended

2. Serge Lang, Undergraduate Analysis (Second Edition). Springer, 2010. ISBN-10: 1441928537

Online Resources:

1. http://rutherglen.science.mq.edu.au/wchen/lnilifolder/lnili.html **Lecture notes by W.W.L. Chen containing an excellent treatment of Lebesgue Integration.**

2. http://www.math.niu.edu/~rusin/known-math/index/28-XX.html **Measure and integration resources from the Mathematical Atlas.**