COURSE NAME: Basics of Metric and Topological Spaces

COURSE CODE: MATH 3402

LEVEL: III

NUMBER OF CREDITS: 3

SEMESTER: II

PREREQUISITES: MATH2401

RATIONALE:

Topology can be thought of as a kind of generalization of Euclidean geometry, and also as a natural framework for the study of continuity. One noticeable feature of theoretical physics over the last decades has been the rapid growth of the use of topological and geometrical methods in condensed matter physics, statistical mechanics, elementary particle physics, general relativity and string theory. Providing students with basic knowledge in Metric and Topological Spaces will prepare them for advanced work in mathematics as well as in other sciences.

COURSE DESCRIPTION:

This course will provide students with a basic knowledge of metric and topological spaces. Furthermore, topological properties such as connectedness and compactness will be explored in detail.

LEARNING OUTCOMES:

At the end of the course, students should be able to:

- Explain the characteristics of metric and topological spaces;
- Classify topological spaces;
- Construct continuous functions between topological spaces;
- Test convergence of sequences in different spaces;
- Identify connections between analysis (real and complex) and topology;
- Recognize when a topological space is connected;
- Verify when a topological space is compact

COURSE CONTENT:

Metrics:

Definition and examples, open neighbourhoods, continuity via neighbourhoods, neighbourhoods and convergence in metric spaces, limits, Cauchy sequences, completeness.

Topology:
Definition of a topology, metric topologies, examples, continuous functions and closed sets, homeomorphisms, topological and non-topological properties, subspaces, product and, Hausdorff spaces.

Compactness:
Definition using open sets, examples, the compact subsets of the real line, continuous images of compact sets, quotient spaces, continuous real valued functions on a compact space, the product of two compact spaces, the compact subsets of Euclidean space, sequential compactness.

Connectedness:
Definition using open sets and integer valued functions, examples, components, path-connectedness.

TEACHING METHODOLOGY
The abstract concepts, illustrated with examples, will be presented during interactive lectures. However, the course is designed in such a way to maximize the extent to which students discover the main concepts by themselves. This can be achieved through class participation in discussions during the lectures and tutorial periods guided by the students themselves and supervised by the lecturer. The advantage is that students feel ownership of the ideas they have worked through themselves. Homework assignments will be divided into two types: *practice problems,* which reinforce the basic concepts and are essentially routine and *challenging problems*, whose resolution will be fundamentally more involved. The total estimated 39 contact hours may be accounted for as follows: 26 hours of lectures and 13 hours of tutorials. Course material, including practice problems, will be posted on the webpage http://ourvle.mona.uwi.edu/

ASSESSMENT

The course assessment has three components:

1. One in-course tests (1h)– 20% of overall grade;
2. Two homework assignments - 20% of overall grade (10% of overall grade each);
3. Final examination (2h) - 60% of overall grade.

REFERENCE MATERIAL:

Books:

Prescribed:

Sutherland, W.A. (2009). *Introduction to Metric & Topological Spaces*. Oxford University Press. ISBN 0198531613

Highly recommended:

Kolmogorov, A. N. & Fomin S. V. (1999). *Elements of the Theory of Functions and Functional Analysis*. Dover Books on Mathematics. ISBN-10: 0486406830

Recommended:

Mendelson, B. (1990): *Introduction to Topology*, Dover, ISBN 0486663523

Jaenich, K. (2001) *Topology*, Springer Undergraduate Texts in Mathematics, 2001 ISBN-10: 0387908927

Gamelin, T. & Greene, R. (1999): *Introduction to Topology*, Dover, ISBN-10: 0486406806

Online Resources:

http://at.yorku.ca/topology/educ.htm
This website presents a well-developed, new and growing collection of notes for students learning topology.

http://www.geom.uiuc.edu/zoo/

The Topological Zoo is an ongoing project which is primarily the work of graduate students from at the University of Minnesota.