COURSE NAME AND CODE: Some Topics in Functional Analysis

COURSE CODE: MATH 3403

LEVEL: III

NUMBER OF CREDITS: 3

SEMESTER: II

PREREQUISITES: MATH2401

RATIONALE:

Functional analysis can be considered as a branch of mathematical analysis, the core of which is formed by the study of vector spaces provided with the limit-related structure (e.g. inner product, norm, and topology) and the linear operators acting upon these spaces and respecting these structures in a suitable sense. The historical roots of functional analysis lie in the study of spaces of functions. The methods of functional analysis turned out to be extremely useful for the study of differential and integral equations.

COURSE DESCRIPTION:

This is a Level III course for majors and minors in Mathematics, which is suitable for mathematics and physics students. This course will give students the basic knowledge of Banach and Hilbert spaces, illustrated on the examples of finite dimensional vector spaces and functional spaces.

LEARNING OUTCOMES:

By the end of the course, students will be able to:

- Classify different norms on finite dimensional spaces, illustrate geometrically;
- Prove the triangular inequalities for different norms in finite dimensional spaces;
- Construct a norm by the inner product in Hilbert space;
- Prove the parallelogram rule for Hilbert space;
- Apply classical inequalities to prove triangular property of the norm;
- Identify several functional spaces and equipped with norms or inner products;
- Give examples of linear operators and evaluate their norm;

CONTENT

Normed vector spaces:
Metric Spaces; Definition and examples of normed vector spaces, H¨older and Minkovkii inequalities; Completeness, Banach Space; finite dimensional vector spaces, C[a,b], Lp, lp spaces.

Hilbert spaces:

Definition of inner product, properties; Hilbert space, connection to Banach and metric spaces; examples, Orthogonality, Cauchy-Schwartz inequality, Parallelogram rule; Theorem of Pythagoras; Bessels inequality.

Linear functionals:
Definition of linear functional, properties; Theorem of Hahn-Banach (real version); examples;

Linear Operators:

Linear operators: examples; Continuous and bounded operators, Norm of operator, Space of operators.

TEACHING METHODOLOGY

The abstract concepts, illustrated with examples, will be presented during the lectures. The total estimated 39 contact hours may be accounted for as follows: 26 hours of lectures and 13 hours of tutorials. Course material, including practice problems, will be posted on the webpage

http://ourvle.mona.uwi.edu/

ASSESSMENT:

The course assessment has three components:

1. One in-course test (1 hour) - 20% of overall grade.
2. Two assignments - 10% each totalling 20% of overall grade.
3. Final examination - 60% of overall grade.

The final examination will be two hours in length.

REFERENCE MATERIAL:

Prescribed:

LANG, S. *Undergraduate Analysis (Undergraduate Texts in Mathematics).* Second Edition, Springer, 2010, ISBN-10: 1441928537.

Highly Recommended:

KOLMOGOROV, A.N. & S.V. Fomin. *Elements of the Theory of Functions and Functional Analysis*, *(1999),* ISBN-10: 0486406830.

Recommended:

EIDELMAN, Juli**,** Vitali Milman & Antonis Tsolomit. *Functional Analysis: An Introduction (Graduate Studies in Mathematics), Volume 66*, American Mathematical Society, 2004. ISBN 0821836463

KREYSZIC, E. *Introductory Functional Analysis with Applications*, John Wiley and Sons, (*1978*)
These books are pedagogically sound, comprehensively address all element of the syllabus, and provide useful case studies and examples.

Online Resources:

1. http://mathforum.org/library/topics/ functional_a/ - The MathForum Internet Mathematics Library is a curated list of online resources for Functional Analysis, including online lecture notes and practice problems. The site is maintained by the Goodwin College of Professional Studies at Drexel University.