COURSE NAME: Introduction to Differential Geometry with Computer Software
COURSE CODE: MATH3404
LEVEL: III
SEMESTER: II
NUMBER OF CREDITS: 3

PREREQUISITES: MATH 2410, MATH2403

RATIONALE:

Curves and surfaces are objects that everyone can see, and many of the questions that can be asked about them are natural and easily understood. Differential geometry presents the main results in the geometry of curves and surfaces in three-dimensional Euclidean space. Prerequisites are kept to an absolute and the most direct and straightforward approach will be used throughout this course.

COURSE DESCRIPTION:

This is a Level III course for majors and minors in Mathematics, which is suitable for mathematics, physics and engineering students with a specialization in robotics. This course will give students the basic knowledge of the geometry of curves and surfaces. Furthermore, properties such as orientability, curvature, torsion and geodesics will be treated in detail.

LEARNING OUTCOMES:

At the end of the course, students will be able to:

- Explain rigorously the main characteristics of curves and surfaces in three-dimensional space;
- Classify quadratic forms and relate them to surfaces;
- Compute geodesics in three-dimensional space;
- Compute curvatures of surfaces;
- Identify connections between linear algebra and multivariable calculus and differential geometry;
- Investigate singularities of vector fields;
- Verify when surfaces are isometric.

CONTENT:

Introduction:
Curves and arc-length, parameterization of curves, closed curves, level curves, curvature, plane curves, space curves.

Global properties of curves:
Simple closed curves, the isoperimetric inequality, the four vertex theorem.

Surfaces in three dimensions:
Smooth surfaces, smooth maps, tangent, normals and orientability. Examples of surfaces: level surfaces, quadratic forms, surfaces of revolution, compact surfaces, triply orthogonal systems. The inverse function theorem and its applications.

The first and second fundamental forms:
Length of curves on surfaces, isometries of surfaces, conformal mappings of surfaces, equiareal maps and a theorem of Archimedes. The second fundamental form, the Gauss and Weingarten maps, curvature of curves on surfaces, normal and geodesic curvature, parallel transport and covariant derivatives.

Lab component
Representation of surfaces and computation of curvature, torsion, geodesics, etc with computer software.

TEACHING METHODOLOGY
The abstract concepts, illustrated with examples, will be presented during the lectures. The course is designed in such a way to maximize the extent to which students discover the main concepts by themselves. This is achieved through class participation in discussions during the lectures and tutorial periods. Homework problems will be divided into two types: *practice problems*, and *challenging problems*, whose resolution will be fundamentally more involved. The total estimated 39 contact hours may be accounted for as follows: 26 hours of lectures and 13 hours of tutorials involving the use of compuer software such as Maple, Mathematica or Mathlab.

Course material, including practice problems, will be posted on the webpage
http://ourvle.mona.uwi.edu/

ASSESSMENT
The course assessment has three components:

- In-course test - 20% of overall grade;
- One group project - 20% of overall grade;
- Final exam - 60% of overall grade.

The final exam will be two hours in length and consist of compulsory questions.

REFERENCE MATERIAL:
Prescribed books:

A. Pressley: *Elementary differential geometry*, Springer Undergraduate Mathematics series, London, 2001

Recommended books:

- B. O'Neill: *Elementary Differential Geometry*, Academic Press, 2006
- J. Oprea: *Differential Geometry and its Applications*, Pearson, 2007

Online Resources:
http://www.trillia.com/online-math/geometry.html
This website presents a list of online lecture notes in differential geometry.
http://www.math.niu.edu/~rusin/known-math/index/53-XX.html

There the student will find additional explanation and different perspectives to the topics treated in the course differential geometry.