COURSE NAME: Advanced Abstract Algebra
COURSE CODE: MATH3411
LEVEL: III
SEMESTER: II
NUMBER OF CREDITS: 3
PREREQUISITES: MATH2411

RATIONALE: Abstract Algebra is one of the twin pillars of modern mathematics. (The other one is Mathematical Analysis.) It has proved to be central to much of mathematics itself and has important applications in other disciplines such as chemistry and physics. This course will provide essential background in one of the areas essential for a modern mathematics major.

COURSE DESCRIPTION: This is a third-year course required for all mathematics majors. It extends students' background knowledge of basic group theory into rings and fields. In ring theory, this course introduces the basic substructures of rings: subrings, ideals, and radicals. It also studies mappings between rings that preserve algebraic structure. In field theory, this course introduces subfields, finite extension fields, and other topics related to the solvability of polynomial equations.

LEARNING OUTCOMES: At the end of the course, students will be able to:

1. Determine whether a particular subset of a ring *R* is a subring, ideal, or radical;
2. Prove elementary facts about subrings and ideals from the relevant definitions and other elementary facts;
3. Prove basic relationships between subrings and ideals (*e.g.*, all ideals are subrings but not all subrings are ideals);
4. Determine whether a ring *R* is an integral domain and whether it is a Euclidean ring;
5. Determine whether a mapping between rings is a homomorphism and, if so, identify its kernel; determine whether two rings are isomorphic;

6. Determine the degree of the extension E of a field F;
7. Prove elementary facts about extension fields from the relevant definitions and other elementary facts;
8. Construct extensions to a field F that contain the roots of a polynomial in $F[x]$

CONTENT:

- **Rings:** *Definition of a ring; classification of rings; elementary facts about rings; homomorphisms between rings; ideals and quotient rings; maximal ideals.*

- **Special types of rings:** Integral domains; elementary facts about integral domains; Euclidean rings; primes in a Euclidean domain; the g.c.d. in a Euclidean domain; the Euclidean algorithm. The rings ***R****[x}* and ***C****[x]*.

- **Field Theory:** Definition and examples of fields; extension fields, the degree of an extension; roots of polynomials; finite fields.

TEACHING METHODOLOGY This course will be delivered by a combination of lectures and tutorials. The total of 39 contact hours is broken down as follows: 28 hours of lectures and 11 hours of tutorials. The tutorial will be interspersed with the lectures by having students discuss exercises, revise material as needed, and cover new content each day. Course materials such as exercises, assignments, solutions, etc., will be posted on the webpage http://ourvle.mona.uwi.edu/

ASSESSMENT The course assessment will be as follows:

i. Three written assignments worth 5% each (15% total)
ii. One (1 hour) in-course examination worth 15%
iii. One (two hours) final examination worth 70%

COURSE TEXTS:

Prescribed: I.N. Herstein: Abstract Algebra, Macmillan, 3rd Edition, 1996, ISBN-10: 0471368792

Highly recommended: I.N. Herstein, Topics in Algebra, John Wiley and Sons, 2nd Edition, 1975 ISBN-10: 0471010901

Recommended: P.M. Cohn: An Introduction to Ring Theory, Springer Undergraduate Series, 2001 ISBN-10: 1852332069

ONLINE RESOURSES:

1. http://ocw.mit.edu/courses/mathematics/18-702-algebra-ii-spring-2011/ - This website presents a well-developed, new and growing collection of notes and visual applications of topics in Abstract Algebra.
2. http://www.jmilne.org/math/CourseNotes/ - Here students will find a variety of downloadable pdf files that should prove useful for this course.