Course Title: **Advanced Linear Algebra**

Course Code: **MATH3412**

Level: **III**

Semester: **I**

No. of credits: **3**

Prerequisites: **A First Course in Linear Algebra (MATH2410)**

Rationale: Linear algebra (including matrix theory) is one of the most widely applied branches of mathematics. It is a significant tool of analysis in pure and applied mathematics and in almost all other branches of the physical and social sciences as well. An understanding of its basic concepts and algorithms will benefit many students in their subsequent courses and, after graduation, in their employment or postgraduate programmes of studies.

Course description: This is a Level III course required for all mathematics majors. It is also expected to be of interest to students in related disciplines. Our primary interest is in the matrix representation of a linear mapping from a finite—dimensional inner-product space to itself. We emphasize throughout the nature and purpose of similarity transformations in representing these mappings with matrices.

Learning outcomes: At the end of this course, successful students will be able to:

1. Define the abstract notions of vector space and inner product space;
2. Understand the linear transformation, kernel and range of a transformation, matrix representation of linear transformation, isomorphism of vector spaces, invertible transformation, transition matrices and similarity;
3. Distinguish the separate roles of matrices: as agents of change of basis and as representing linear transformations;
4. Define the adjoint of an operator;
5. Explain when eigenvalues exist for a given operator;
6. State the spectral theorem;
7. Understand how to find the singular-value decomposition of an operator;
8. Understand the idea of Jordan blocks, Jordan matrices, and the Jordan form of a matrix;

Content:

Vector Spaces: Vector spaces over an arbitrary field, subspaces of vector spaces, span and independence, bases and finite dimensional vector spaces, bases and infinite dimensional vector spaces, coordinate vectors.

Linear Transformation: Short introduction to linear transformations, range and kernel, correspondence and isomorphism theorems, matrix representation, algebra of L(V,W) and $M_{mn}(F)$, invertible transformations and matrices.

Theory of linear operators: invariant subspaces, cyclic operators, maximal vectors, indecomposable linear operators, canonical form, operators on real and complex vector spaces. **Inner product spaces:** inner product, geometry in inner product spaces, orthonormal sets and the Grahm-Schmidt process, orthogonal complements and projections, dual spaces, adjoints.

Linear operators on inner product spaces: self-adjoint and normal operators, spectral theorems, unitary and orthogonal operators, polar decomposition and singular value decomposition, trace of a linear operator.

Bilinear maps and forms: basic properties, symplectic spaces, quadratic forms and conic sections, Jordan canonical form.

Teaching methodology: This course will be delivered by a combination of lectures and tutorials. The total of 39 contact hours is broken down as follows: 28 hours of lectures and 11 hours of tutorials. The tutorial will be interspersed with the lectures by having students discuss exercises, revise material as needed, and cover new content each day. Course materials such as exercises, assignments, solutions, etc., will be posted on the webpage

http://ourvle.mona.uwi.edu/

Assessment: The assessment will be based on the following:

1. Four written assignments worth 5% each (20% total) and one incourse test worth 20%;
2. One Final Examination (2 hours) worth 60%.

Course texts:

Prescribed

1. Cooperstein D., Advanced Linear Algebra, CRC Press, 2010. ISBN-10: 1439829667

Highly recommended

2. Gelfand I. M., Lectures on linear algebra, Dover, 1989. ISBN-10: 0486660826
3. Halmos P. R.., Finite-dimensional vector spaces, Springer Verlag, 1987, ISBN 0387900934

Recommended

1. Lax P. D., Linear Algebra, John Wiley & Sons, 1997. ISBN 0471111112
2. Davis H. T. and Thoms K. T., Linear Algebra and Linear Operators in Engineering, Academic Press, 2000, ISBN-10: 012206349X

Online Resources:

1. http://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/related-resources/ - This website presents a well-developed, new and growing collection of notes and visual applications of topics in linear and advanced linear algebra.
2. http://math.utoledo.edu/~codentha/Linear_Algebra/index.html - Here some elementary and more advanced texts on linear algebra can be downloaded. These texts are freely available online.