Course Title:	TOPICS IN MATHEMATICAL ANALYSIS
Course Code:	MATH6624
# of Contact hrs.:	One Semester (13 weeks - 36 hours of lectures and 24 hours of tutorials)
Semester:	I
No. of credits:	4
Prerequisites:	None

Rationale:

Even though the students are familiar with the notion of limit from high school, they still need to understand deeper the limit concept, countability and non-countability, consider paradoxes and counter examples, extend their understanding of convergence of sequences and series of functions, derivative, integral.

Aim:

The aim of the course is to teach students the tools of modern analysis as it related to further study in mathematics, especially numerical analysis, differential equations, functional analysis, topology, stochastic processes. This course is intended to develop the ability of the students to work with abstract ideas.

Learning Outcomes:

- By completion of this course, students should be able to:
- Define infimum and supremum of the set on real line;
- Find infimum and supremum of selected sets;
- Explain the role of paradoxes in the set theory;
- State and prove Bolzano-Weierstass and Heine-Borel theorems;
- Define the Riemann-Stieltjes integral and explain the difference with the Riemann integral;
- Explain the notion of compactness and give example of compact and non-compact sets;
- Explain the notion of point-wise and uniform convergence of functions and give examples.

Teaching Methods:

Students will be exposed to the theoretical aspects of measure and integration via lectures. Tutorials and group problem solving sessions will be held regularly to ensure that students take an active role in the development of concepts encountered on the course, and are able to apply those concepts in the construction of mathematical proof.

The total estimated 48 contact hours may be accounted for as follows: 36 hours of lectures and 24 hours of tutorials. The tutorial classes will include problem solving sessions which will ensure that the students are able to understand, appreciate and apply the concepts learnt in the course; and computer laboratory sessions where they will learn to handle large data sets, parallel computing and to implement theoretical/numerical schemes learnt in the lecture hours. Course material, including note sets and exercises will be posted on the webpage

http://ourvle.mona.uwi.edu/

Syllabus

Real numbers and Set equivalence [8 hrs i.e. 6 hrs lecture + 4 hrs tutorials]

Axioms of real numbers, infimum, supremum, integers, rationals. Set equivalence, countable and uncountable sets, axiom of choice, Godel and independence proofs, Zorn' lemma, Zermelo-Russell paradox.

Sequences of real numbers: [8 hrs i.e. 6 hrs lecture + 4 hrs tutorials]

Convergent and divergent sequences, limit theorems, number e and real exponents, Bolzano-Weierstrass Theorem, Cauchy condition, limsup and liminf of bounded and unbounded sequences.

Calculus for Vector-Valued functions: [6 hrs i.e. 4 hrs lecture + 4 hrs tutorials]

Vector-valued functions, limits and continuity, differentiation and Jacobian, inverse function theorem, implicit function theorem. Compactness in R^n. The Heine-Borel theorem and the consequence for continuos function on the Real line.

Sequences and series of functions: [12 hrs i.e. 8 hrs lecture + 8 hrs tutorials]

Pointwise and uniform convergence of functions, integration and differentiation of uniformely convergent sequences.

Series of functions, Weiershtrass Test, Weierstrass' example. Summability methods and Tauberian theorems.

Riemann-Stieltjes integral 14 hrs i.e. 12 hrs lecture + 4 hrs tutorials]

Riemann-Stieltjes integration with respect to an increasing integrator, Riemann-Stieltjes sums, Riemann-Stieltjes integration with respect to an arbitrary integrator, functions of bounded variations, Riemann-Stieltjes integration with respect to functions of bounded variation, measure zero and Cantor set, necessary and sufficient conditions for existence of Riemann integral.

Total contact hrs: 48 i.e. 36 hrs lecture + 24 hrs tutorials

Assessment

The course assessment has two components consisting of coursework (40%) and final exam (60%)

One In-course test – 30% of overall grade;

One Written assignment – 10% of overall grade;

Final exam (3 hours) – 60% of overall grade.

Reference Material

Text Books:

Johnsonbaugh, R. & W. E. Pfaffenberger. (2010). *Foundations of Mathematical Analysis*, Dover, ISBN-10: 0486477665

Garrity, T. A. (2008). *All the Mathematics You Missed (But need to Know for Graduate School*), Cambridge University Press, 2008, ISBN-10: 0521797071

Marsden, J. E. *et al.* (1974). *Elementary Classical Analysis*. W.H. Freeman and Company. ISBN-10: 0716721058

Muresan, M. (2000). *A Concrete Approach to classical Analysis*, Springer, ISBN-10: 1441927050

Bridger M. (2007). *Real Analysis: A Constructive Approach,* J. Wiley & Sons. ISBN-10: 0471792306

Online Resources:

1) http://www.mathcs.org/analysis/reals/

Interactive Real Analysis is an online, interactive textbook for Real Analysis or Advanced Calculus in one real variable. It deals with sets, sequences, series, continuity, differentiability, integrability (Riemann and Lebesgue), topology, power series, and more.

2) http://www.jirka.org/ra/

E-book giving an introduction to Real Analysis

3)https://files.nyu.edu/jmg336/public/html/mathematics.html#analysis for links to everything from lecture notes, videos, tutorials, and on-line texts regarding Real Analysis.