Course Name:	MEASURE AND INTEGRATION
Course Code:	MATH6625
# of CONTACT HRS:	One Semester (13 weeks -36 hours of lectures and 24 hours of tutorials)
Credits:	4
Level:	Graduate
Pre-requisite:	None

RATIONALE

The construction of the Riemann integral is known to every undergraduate mathematics student. However, this integral has certain inherent limitations that prevent its use in a wide range of mathematical applications, including modern developments in the fields of probability theory, stochastic processes and dynamical systems. This issue is dealt with by extracting the fundamental ideas behind the construction of such integrals, and providing a mathematically rigorous, though abstract, foundation upon which to define more powerful notions of integration.

AIM

We consider the limitations of the Riemann integral, and show that it is necessary to develop a precise mathematical notion of `length' and `area' in order to overcome them. Thus we develop the more abstract concept of measure, and use it to construct the more Lebesgue integral, and to investigate its properties. Finally we look at the role played by measure and Lebesgue integration in modern probability theory. This course is intended to develop the ability of students to work with abstract ideas.

LEARNING OUTCOMES

On completion of this course, the student should be able to:

- Define the Dirichlet function and explain why it is not Riemann integrable;
- Demonstrate the role of Lebesgue outer inner measure in the definition of measurable sets;
- Prove the Caratheodory Criterion for measurability;

- Use the Caratheodory Criterion to prove selected properties of measurable sets;
- Evaluate the Lebesgue integral of selected functions;
- State and prove the monotone convergence theorem, dominated convergence theorem, Fatou's lemma;
- Explain the role of each in the construction of the Lebesgue integral;
- Describe the elementary approach to probability and explain why it fails for general sample spaces;
- Explain how the Lebesgue measure may be used to construct a probability space.

TEACHING METHODOLGY:

Students will be exposed to the theoretical aspects of measure and integration via lectures. Tutorials and group problem solving sessions will be held regularly to ensure that students take an active role in the development of concepts encountered on the course, and are able to apply those concepts in the construction of mathematical proof.

The total estimated 48 contact hours may be accounted for as follows: 36 hours of lectures and 24 hours of tutorials. The tutorial periods will include problem solving sessions which will ensure that the students are able to understand, appreciate and apply the concepts learnt in the course. Course material, including note sets and exercises will be posted on the webpage

http://ourvle.mona.uwi.edu/

SYLLABUS

Introduction [4 hrs. i.e. 3 hrs lecture + 2 hrs tutorials]: First principles construction of the Riemann integral; Riemann integrable functions; the role of the Fundamental Theorem of Calculus; the Dirichlet function.

Lebesgue Measure [12 hrs. i.e. 9 hrs lecture + 6 hrs tutorials]: Measure in the line and plane; abstract measure spaces and the Caratheodory criterion; properties of Lebesgue measure; construction of non-measurable sets.

Measurable functions [12 hrs i.e. 9 hrs lecture + 6 hrs tutorials]: Measurable functions on R; simple functions; algebraic operations on measurable functions; equivalent functions.

The Lebesgue integral [12 hrs. i.e. 9 hrs lecture + 6 hrs tutorials]: Definition and basic properties on bounded sets; the Monotone Convergence Theorem; Fatou's Lemma; the Dominated Convergence Theorem; integration over sets of infinite measure; comparative analysis of Riemann and Lebesgue integrals; generalisation of the Fundamental Theorem of Calculus.

Applications to probability theory [8 hrs i.e. 6 hrs lecture + 4 hrs tutorials] Probability spaces as measure spaces; random variables as measurable functions; expectation as Lebesgue integral with respect to a probability measure.

Total contact hrs = 48 i.e. 36 hrs lecture + 24 hrs tutorials

ASSESSMENT:

The course assessment has two components consisting of coursework (40%) and a final exam (60%):

One in-course test – 20% of overall grade;

One Written assignment – 20% of overall grade;

Final exam – 60% of overall grade will be three hours duration.

REFERENCE MATERIAL

Books:

Bear, H.S. (2002). *A Primer of Lebesgue Integration (Second Edition).* Academic Press, ISBN-10: 0120839717

Strook, D. W. (1998). A Concise Introduction to the Theory of Integration (Third Edition), Birkhauser Boston, ISBN-10: 0817640738.

N.B *[These textbooks are excellent for the graduate course, it is available online on ScienceDirect, and the second edition is the latest edition.*

Integration theory doesn't have a constant flow of new textbooks as other subjects might and as such an exception is being requested in this case for books 5 years and younger.]

Online resources:

http://ocw.mit.edu/courses/mathematics/18-125-measure-and-integration-fall-2003/index.htm: MIT open courseware page for the course “Measure and Integration”. Includes lecture notes;