COURSE NAME:	ELEMENTS OF FUNCTIONAL ANALYSIS
COURSE CODE:	MATH6626
# of CONTACT HRS:	One semester (13 weeks - 36 hours of lectures and 24 hours of tutorials)
NUMBER OF CREDITS:	4
LEVEL:	Graduate
PREREQUISITES:	MATH6621

RATIONALE:

Functional analysis is a branch of mathematical analysis dealing with the study of normed, Banach, and Hilbert spaces endowed with some kind of limit-related structure such as for instance an inner product, norm, topology, etc. and the linear operators acting upon these spaces and respecting these structures in a suitable sense. The historical roots of functional analysis lie in the study of spaces of functions and the formulation of properties of transformations of functions such as the Fourier transform as transformations defining continuous, unitary, etc. operators between function spaces. This point of view turned out to be particularly useful for the study of differential and integral equations.

COURSE DESCRIPTION:

The course gives an up-to-date and modern overview of the main concepts in Functional Analysis. Functional analytic properties and several examples of Banach and Hilbert spaces arising in several branches of mathematics are studied to show how functional analysis is a unifying theme in different mathematical fields.

LEARNING OUTCOMES:

At the end of the course, students will be able to:

- Explain rigorously the meaning of metric, normed, Banach, and Hilbert spaces;
- Discuss the implications of the Hahn-Banach theorem, Cathegory theorem, Uniform Boundedness Theorem, Open Mapping Theorem, Closed Graph Theorem;
- Construct functionals and adjoints;
- Construct operators between normed, Banach, and Hilbert spaces;
- Test strong and weak convergence of sequences in different spaces;

- Test convergence of sequences of operators and functionals;
- Identify connections between functional analysis and topology;
- Use examples to explain the significance of Hahn-Banach theorem, Cathegory theorem, Uniform Boundedness Theorem, Open Mapping Theorem, Closed Graph Theorem.

CONTENT:

Metric spaces: metric space, examples of metric spaces, open sets, closed sets, neighborhood, convergence, Cauchy sequence, completeness. [5 hrs i.e. 4 hrs lecture + 2 hrs tutorials]

Normed spaces: vector space, normed space, Banach space, properties of normed spaces, finite dimensional normed spaces and subspaces, compactness and finite dimension. [5 hrs i.e. 4 hrs lecture + 2hrs tutorials]

Linear operators: linear operators, bounded and continuous linear operators between Banach spaces, linear functionals, linear operators and functionals on finite dimensional spaces, normed spaces of operators, the dual space. [12 hrs i.e. 8 hrs +8 hrs tutorials]

Hilbert spaces: inner product space, Hilbert space and its properties, orthogonal complements and direct sums, orthonormal sets and sequences, representation of functionals on Hilbert spaces, adjoint, self-adjoint, unitary and normal operators, strong and weak convergence, convergence of sequences of operators and functionals. [16 hrs i.e. 12 hrs lecture + 8 hrs tutorials]

Some fundamental theorems of Functional Analysis: Zorn's lemma, Hahn-Banach theorem, Cathegory theorem, Uniform Boundedness Theorem, Open Mapping Theorem, closed linear operators and the Closed Graph Theorem. [10 hrs i.e. 8 hrs lecture + 4 hrs tutorials]

Total contact hrs = 48 i.e. 36 hrs lecture + 24 hrs tutorials

TEACHING METHODOLOGY

The abstract concepts, illustrated with examples, will be presented during the lectures. The course is designed in such a way to maximize the extent to which students discover the main concepts by themselves. This is achieved through class participation in discussions during the lectures and tutorial periods. The tutorial periods will include problem solving sessions which will ensure that the students are able to understand, appreciate and apply the concepts learnt in the course; and computer laboratory sessions where they will learn to handle large data sets, parallel computing and to implement theoretical/numerical schemes learnt in the lecture hours. Homework problems will be divided into two types: *practice problems*, and *challenging problems*, whose resolution will be fundamentally more involved. The total estimated 48 contact hours may be accounted for as follows: 36 hours of lectures and 24 hours of tutorials.

Course material, including practice problems, will be posted on the webpage

http://ourvle.mona.uwi.edu/

ASSESSMENT

The course assessment has two components consisting of coursework (40%) and a final exam (60%):

One in-course test - 20% of overall grade;

One group project - 20% of overall grade;

Final exam 3 hrs - 60% of overall grade.

The final exam will be three hours in duration.

REFERENCE MATERIAL:

<u>*Prescribed books:*</u>

SEN, R. (2013. *A first course in Functional Analysis*, Anthem Press ISBN-10: 0857283200

<u>*Recommended books:*</u>

Reed M. & B. Simon (1980). *Methods of Modern Mathematical Physics, vol. 1: Functional Analysis*, Academic Press, ISBN-10: 0125850506

Szekeres P. (2004). *A course in Modern Mathematical Physics: Groups, Hilbert Space, and Differential Geometry*, Cambridge University Press, ISBN-10: 0521829607

Teschl, G. (2005). *Functional Analysis*. This book can be freely downloaded from the homepage of Prof. Teschl at the University of Wien, Austria following the link http://www.mat.univie.ac.at/~gerald

Online Resources:

http://www.freebookcentre.net/Mathematics/Functional-Analysis-Books.html
This section contains free e-books and guides on Functional Analysis, some of the resources in this section can be viewed online and some of them can be downloaded.

http://www.mat.univie.ac.at/~gerald/ftp/book-fa/index.html

This manuscript provides a brief introduction to Real and Functional Analysis. It covers basic Hilbert and Banach space theory as well as basic measure theory including Lebesgue spaces and the Fourier transform.