

COURSE NAME:	GROUP THEORY WITH APPLICATIONS
COURSE CODE:	MATH6627
# of CONTACT HRS:	Semester I (13 weeks -36 hours of lectures and 24 hours of tutorials)
NUMBER OF CREDITS:	4
LEVEL:	Graduate
PREREQUISITES:	None

RATIONALE:

Group theory studies the algebraic structures known as groups. The concept of a group is central to abstract algebra and recurs throughout mathematics. Moreover, the methods of group theory have strongly influenced many parts of algebra. Lie groups theory is a branch of group theory that has experienced tremendous advances and it cannot be skipped in any course in group theory. Group theory and the closely related representation theory have many applications in physics and chemistry since various physical systems, such as crystals and the hydrogen atom, can be modeled by symmetry groups.

COURSE DESCRIPTION:

The course gives an up-to-date and modern overview of the main concepts in Group Theory. Group theoretical properties and several examples of groups arising in many branches of mathematics, physics, and chemistry are studied to show how group theory emerges in different mathematical fields and in applied sciences.

LEARNING OUTCOMES:

At the end of the course, students will be able to:

- Explain rigorously the meaning of transformation, symmetry, permutation, Lie groups;
- Discuss the implications of symmetry groups in physics and chemistry;
- Construct quaternions and Clifford algebras;
- Identify Lie groups and Lie algebras;
- Construct one-parameter subgroups;

- Identify group of isometries;
- Use examples to explain the significance of Grassmann algebras.

CONTENT:

Groups: Elements of group theory, transformations and permutation groups, matrix groups, normal subgroups and factor groups, group actions. [11 hrs i.e. 8 hrs lecture + 6 hrs tutorials]

Symmetry groups: Rotation group $SO(3)$, Euclidean group, Galilean group, Lorentz group.[5 hrs i.e.4 hrs lecture + 2 hrs tutorials]

Algebras: Algebras and ideals, complex numbers and complex structures, quaternions and Clifford algebras, Grassmann algebras. [11 hrs i.e. 8 hrs lecture + 6 hrs tutorials]

Lie groups and Lie algebras: Lie groups, matrix Lie groups, infinitesimal generators, the exponential map, one-parameter subgroups, complex Lie groups, Lie groups of transformations, group of isometries. [21 hrs i.e. 16 hrs lecture + 10 hrs tutorials]

Total contact hrs 48 i.e. 36 hrs lecture+24 hrs tutorials

TEACHING METHODOLOGY

The abstract concepts, illustrated with examples, will be presented during the lectures. The course is designed in such a way to maximize the extent to which students discover the main concepts by themselves. This is achieved through class participation in discussions during the lectures and tutorial periods. The tutorial periods will include problem solving sessions which will ensure that the students are able to understand, appreciate and apply the concepts learnt in the course; and computer laboratory sessions where they will learn to handle large data sets, parallel computing and to implement theoretical/numerical schemes learnt in the lecture hours. Homework problems will be divided into two types: *practice problems*, and *challenging problems*, whose resolution will be fundamentally more involved. The total estimated 48 contact hours may be accounted for as follows: 36 hours of lectures and 24 hours of tutorials.

Course material, including practice problems, will be posted on the webpage

<http://ourvle.mona.uwi.edu/>

ASSESSMENT

The course assessment has two components consisting of coursework (40%) and a final exam (60%):

One in-course test - 20% of overall grade;

One group project - 20% of overall grade;

Final exam - 60% of overall grade will be three hours duration.

REFERENCE MATERIAL:

Prescribed books:

Costa, G. & G. Fogli (2012). *Symmetries and Group Theory in Particle Physics, Lecture Notes in Physics*, Vol. 283, Springer Verlag. ISBN 978-3-642-15481-2

Recommended books:

Gilbert, W. J. & W. Keith Nicholson (2003), *Modern Algebra with Applications*, John Wiley & Sons, ISBN-10: 0471414514

Stillwell, J. (2008). *Naive Lie Theory*, Springer Verlag. ISBN-10: 144192681X

Sattinger, D. H. & O. L. Weaver (1993), *Lie groups and Algebras with Applications to Physics, Geometry, and Mechanics*, Springer Verlag. ISBN-10: 0387962409

Online Resources:

<http://www.e-booksdirectory.com/listing.php?category=35>

This page lists freely down-loadable books about group theory and Lie groups.