COURSE NAME:	DIFFERENTIAL GEOMETRY
COURSE CODE:	MATH6628
# of CONTACT HRS:	One Semester (13 weeks –36 hours of lectures and 24 hours of tutorials]
NUMBER OF CREDITS:	4
LEVEL:	Graduate
PREREQUISITES:	None

RATIONALE:

Differential geometry is a field in mathematics using several techniques of differential and integral calculus, as well as linear algebra, to study problems in geometry. The theory of plane and space curves and of surfaces in the three-dimensional Euclidean space formed the basis for the development of differential geometry during the past two centuries. Since the late 19th century, differential geometry has grown into a field concerned more generally with the geometric structures on differentiable manifolds.

COURSE DESCRIPTION:

This course is about the analysis of manifolds such as curves, surfaces and hypersurfaces in higher dimensional space using the tools of calculus and linear algebra. There will be many examples discussed, including some which arise in the theory of general relativity. Emphasis will be placed on developing intuitions and learning to use calculations to verify and prove theorems. Students need a good background in linear algebra. Some exposure to differential equations is helpful but not absolutely necessary.

LEARNING OUTCOMES:

At the end of the course, students will be able to:

- Explain rigorously the geometrical meaning of curvature and torsion of curves, surfaces and more complex manifolds;
- Compute Lie derivatives of vector fields and differential forms;
- Compute covariant derivatives of tensor fields;

- Compute geodesics in Riemannian and pseudo-Riemannian manifolds;
- Compute curvatures and scalar curvatures of Riemannian and pseudo-Riemannian manifolds ;
- Investigate isometries of manifolds by solving the corresponding Killing equation;
- Identify connections between linear algebra and multivariable calculus and differential geometry;
- Investigate singularities of manifolds;
- Solve Einstein field equations in vacuum for spherically symmetric manifolds.

CONTENT:

Manifolds: Heuristic introduction, definitions, examples, differentiable maps, tangent and cotangent space, one-forms, tensors and operations with tensors, tensor fields, pullback and pushforward, one-parameter group of transformations, flows and Lie derivatives. [10 hrs i.e. 8 hrs lecture + 4 hrs tutorials]

Differential Forms: Definitions, exterior derivatives, interior product and Lie derivatives of forms, integration of differential forms, orientation, Stokes theorem. [4 hrs i.e. 4 hrs lecture + 2 hrs tutorials]

Riemannian Geometry: Riemannian and pseudo-Riemannian manifolds, metric tensor, parallel transport, connection and covariant derivative, affine connections, parallel transport and geodesics, the covariant derivative of tensor fields, the metric connection. [13 hrs i.e. 10 hrs lecture + 6 hrs tutorials]

Curvature, torsion and Levi-Civita connections: Definitions, geometrical meaning of the Riemann tensor and the torsion tensor, the Ricci tensor and scalar curvature, Levi-Civita connections, the normal coordinate system, Riemannian curvature tensor with Levi-Civita connection. [11 hrs i.e. 8 hrs + 6 hrs tutorials]

Isometries and conformal transformations: Definitions, Killing vector fields, non-coordinate bases, Cartan's structure equations, Levi-Civita connection in a non-coordinate bases. [5 hrs i.e. 4 hrs + 2 hrs tutorials]

Aspects of General Relativity: Axiomatic introduction to general relativity, the Einstein-Hilbert action, Einstein field equations and their solution for a spherically symmetric manifold. [5 hrs i.e. 4 hrs + 2 hrs tutorials]

Total contact hrs = 48 hrs i.e. 36 lecture + 24 tutorials.

TEACHING METHODOLOGY

The abstract concepts, illustrated with examples, will be presented during the lectures. The course is designed in such a way to maximize the extent to which students discover the main concepts by themselves. This is achieved through class participation in discussions during the lectures and tutorial periods. The tutorial periods will include problem solving sessions which will ensure that the students are able to understand, appreciate and apply the concepts learnt in the courses. Homework problems will be divided into two types: *practice problems*, and *challenging problems*, whose resolution will be fundamentally more involved. The total estimated 48 contact hours may be accounted for as follows: 36 hours of lectures and 24 hours of tutorials involving the use of computer software such as Maple, Mathematica or Mathlab.

Course material, including practice problems, will be posted on the webpage

http://ourvle.mona.uwi.edu/

ASSESSMENT

The course assessment has two components consisting of coursework (40%) and a final exam (60%):

One in-course test - 20% of overall grade;

One group project - 20% of overall grade;

Final exam - 60% of overall grade will be three hours duration.

REFERENCE MATERIAL:

Prescribed books:

Oprea, J. (2007) *Differential Geometry and its applications*. Pearson. ISBN-10: 0883857480

Recommended books:

O'Neill, B. (1983). *Semi-Riemannian Geometry with applications to Relativity*, Academic Press, ISBN-10: 0123994616

Oprea J. (2007): *Differential Geometry and its Applications*, Pearson, ISBN-10: 0883857480

Garrity, T. A , et al. (2008). *All the Mathematics You Missed [But Need to Know for Graduate School]*, Cambridge University Press, ISBN-10: 0521797071

Szekeres P. (2004): *A course in Modern Mathematical Physics: Groups, Hilbert Space, and Differential Geometry*, Cambridge University Press, ISBN-10: 0521829607

Online Resources:

http://www.trillia.com/online-math/geometry.html

There the student will find additional text books and lecture notes on the topics treated in this course.