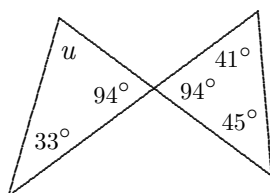


Solutions to Practice Examination

(1) Juan ate one sweetie on Monday, two on Tuesday, three on Wednesday, and so on. This continued until he ate 10 sweeties on the following Wednesday. The total number he ate was $1 + 2 + 3 + \cdots + 10 = 55$.

(2) If a number between 1 and 100 contains the digit 2 exactly once it could be in the units place or the tens place (but not both). There are nine numbers with a 2 in the units but not the tens place. These are 2, 12, 32, 42, 52, 62, 72, 82, and 92. There are nine more numbers with a 2 in the tens but not the units place. These are 20, 21, 23, 24, 25, 26, 27, 28, and 29. Then there are 18 such numbers in all.

(3) Recall that the sum of the angles in any triangle is 180° . Then in the triangle on the right side, the third angle must be 94° . In the triangle on the left side, the corresponding angle is also 94° .

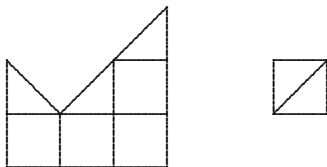


(Vertical angles are equal.) Summing angles in the left triangle, $u + 33^\circ + 94^\circ = 180^\circ$. It follows that $u = 53^\circ$.

(4) The symbols repeat in cycles of four. Then the 4th, 8th, 12th, and so on, up to the 2008th one are all \vee . After that, the 2009th symbol is $<$, the 2010th one is \wedge , and the 2011th one is $>$.

(5) We know that the sweetie is not in the white box and not in the green one. Then the sweetie is in the red box. We also know that the chocolate is either in the red box or the white box. Since the sweetie is in the red box, the chocolate is in the white box. So the green box is empty.

(6) Each triangle is exactly half of a small square. Thus its area is 5. Since the figure consists of four squares and three triangles, its area is $4(10) + 3(5) = 40 + 15 = 55$.



(7) The numbers divisible by both 3 and 4 are the same as the numbers divisible by 12. These are the multiples of 12. The multiples of 12 between 100 and 1000 start with 108, 120, 132, and so on. They continue up to 996. There are 75 such numbers in all.

(8) If 10 marbles are added the weight of the box increases by 150 g. (This is the increase from 650 g to 800 g). Then if 10 marbles are removed the weight of the box decreases by 150 g. Start over with 30 marbles in the box and a total weight of 650 g. Removing 10 marbles leaves a weight of 500 g; removing 10 more leaves a weight of 350 g; removing the last 10 leaves a weight of 200 g. The box weighs 200 g empty.

(9) Each year, Juan's age increases by one year and his sons' combined ages increase by two. Then each year their combined ages get one year closer to Juan's. Since Juan is 33 and his children's combined ages are 21, their ages will be equal to Juan's in 12 years.

(10) Marta wrote 9 numbers with a single digit (the numbers 1 through 9). She wrote 90 numbers with two digits each (the numbers 10 through 99). She wrote 900 numbers with three digits each (the numbers 100 through 999). Finally, she wrote one number (1000) with four digits. The total number of digits she wrote was

$$9(1) + 90(2) + 900(3) + (1)4 = 9 + 180 + 2700 + 4 = 2893$$

(11) To begin, note that the second row must be completed as shown in either figure below. Also, only a 3 or a 4 can go in the small square in the upper-left corner. This gives two possibilities as

3			
1	2	4	3
		3	
			4

4			
1	2	4	3
		3	
			4

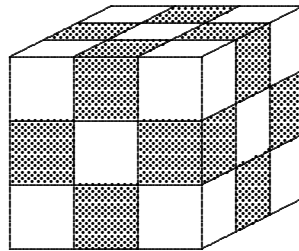
shown above. However, once this choice has been made there will only be one way left to finish each figure. Then there are two ways to complete the original figure, and each one is shown below.

3	4	2	1
1	2	4	3
4	1	3	2
2	3	1	4

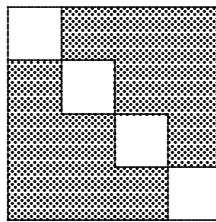
4	3	1	2
1	2	4	3
2	4	3	1
3	1	2	4

(12) There are 4 choices for the first digit: 1, 3, 5, or 7. For each one, there are three more choices for the next digit. (Remember that you cannot use the same digit twice.) For each pair of choices, there are two more possibilities for the last digit. Then there are $4 \times 3 \times 2 = 24$ possible combinations.

(13) The small cubes with exactly two sides painted blue are the ones which were originally in the locations shaded below. There are 4 of them in the top layer, 4 in the middle layer, and 4 in the bottom layer. In all, there are 12 smaller cubes with exactly 2 sides painted blue.



(14) Since the outer square has perimeter 32, each of its sides has length 8. Then the area of the outer square is 64. In addition, each side of a small square has length 2. Then the area of each small square is 4. Therefore, the shaded area is $64 - 4(4) = 64 - 16 = 48$.

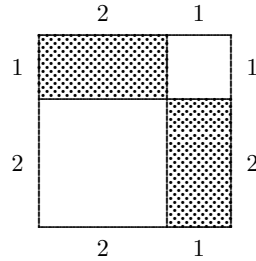


(15) If a rabbit makes 4 jumps in 6 seconds, it will make 2 jumps in 3 seconds. Then it will make 5×2 jumps in 5×3 seconds. In other words, to make 10 jumps it will take 15 seconds.

(16) It is stated that the doctor is the youngest of the three and an only child. Since Cernaldo is not the youngest he is not the doctor. Since Arnolde is not an only child (he has a sister), he is not the doctor either. Then the doctor is Bernardo. Also, the engineer is not Cernaldo and so the engineer must be Arnolde. Finally, the mathematician has to be Cernaldo. The doctor, engineer, and mathematician are B, A, and C in that order.

(17) The largest digit in a *quaddie* cannot be a 1. Otherwise, the number is 111 and the sum of its digits is less than 4. Also, if a number's largest digit is 5, 6, 7, 8, or 9 then the sum of its digits will be more than 4. So we must consider numbers whose largest digit is 2, 3, or 4. If a number's largest digit is 2 its digits could be 2, 1, and 1 or 2, 2, and 0. If a number's largest digit is 3 its digits would be 3, 1, and 0. If a number's largest digit is 4 its digits are 4, 0, and 0. Then the *quaddies* are 112, 121, 211, 202, 220, 103, 130, 301, 310 and 400. There are 10 of them in all.

(18) Based on the information given, the smaller white square is 1×1 and the larger one is 2×2 . Then the outer square is 3×3 . Since the total shaded area is 4 and the outer square has area 9, the shaded proportion is $\frac{4}{9}$.



(19) The total number of chairs is $(6 \times 4) + (4 \times 2) + (3 \times 6) = 24 + 8 + 18 = 50$.

(20) If there is 1 green pencil then there are 6 blue ones and so 13 red pencils in the box. But this is impossible because we know there are fewer red pencils than blue ones in the box. If there are 2 green pencils then there are also 12 blue ones and so 6 red pencils in the box. This is certainly possible. If there are 3 or more green pencils then there are at least 18 blue ones and so at least twenty-one pencils in the box. But this is impossible because we know there are only twenty in the box. Then there are 2 green pencils, 12 blue pencils, and 6 red pencils in the box.