

The University of the West Indies, Mona
presents
The 2022 Senior Mathematics Olympiad

Qualifying Round Examination (SOLUTIONS)

1. **Soln (E)** From -2 to -6 , the number of units is 4. From -6 to 0, the number of units is 6 and from 0 to 5 the number of units is 5 for a total of $4 + 6 + 5 = 15$ units
2. **Soln (A)** The smallest difference occurs when the largest two digit number is subtracted from the smallest three digit number. The result is

$$356 - 87 = 269$$

3. **Soln (B)** The ■ is the 5th, 10th, 15th, ..., 2020th. The 2022nd figure is therefore □
4. **Soln (A)** March has 31 days and so the number of days that span 23th of March to April 23th (inclusive) is

$$(31 - 23 + 1) + 23 = 32$$

Because $1 + 5 \times 7 = 36 (= 32 + 4)$, March 13th will fall on the same day as April $(23 + 4)$ th = April 27th. This is 4 days from Tuesday. It is Saturday.

5. **Soln (C)** Let the amount be \$1. After year one, its value is 0.9. After year 2, $(0.9)^2 = 0.81$. After year 3, $(0.9)^3 = 0.729$. After year 3, $(0.9)^6 = 0.53144$. After year 7, $(0.9)^7 = 0.47830$. This is less than 0.5. The answer is therefore 7 years.
6. **Soln (C)** Suppose x games were played. Since Jimmy won exactly 3 games, he lost $x - 3$ games and Marty won exactly $x - 3$ games and lost 3 games. So

$$\begin{aligned} 2(x - 3) - 3 &= 5 \\ 2x - 6 - 3 &= 5 \\ 2x &= 14 \\ x &= 7 \end{aligned}$$

7. **Soln (D)** In 60 seconds (1 minute) Mother and Father, working together can frost $3 + 2 = 5$ cupcakes and so in 5 minutes, they can frost $5 \times 5 = 25$ cakes.
8. **Soln (B)** First 50 cats are male and 25 cats were accompanied by $25 \times 4 = 100$ kittens. The total number of cats to the shelter is 50 (male) plus 25 (those without litter) plus 25 (those with litters) plus 100 (number in the litters). This sum is 200.

9. **Soln (D)** Let the numbers be x and y with $x < y$. So $xy = 9$ and $\frac{1}{x} = 4 \cdot \frac{1}{y}$. It follows that $4x = y$. Since $4x = y$, $x \cdot 4x = 9$ and so $x^2 = \frac{9}{4}$ and $x = \frac{3}{2}$. Since $x = \frac{3}{2}$, $y = \frac{4 \cdot 3}{2} = 6$ and $x + y = \frac{3}{2} + 6 = \frac{15}{2}$.

10. **Soln (D)** Let x be the number of litres in the tank before the addition

$$x + 15 = \frac{75}{100} \times 28$$

This gives $x + 15 = 21$. So $x = 6$.

11. **Soln (C)** Let the numbers of red and blue marbles be r and b . $\frac{3}{5} = \frac{b}{b+r}$. This gives $5b = 3b + 3r$ or $2b = 3r$ or $b = \frac{3}{2}r$. If the number of red marbles is doubled and the number of blue marbles stays the same, the total is $2r + b$ and the fraction of red marbles is

$$\frac{2r}{2r + b} = \frac{2r}{2r + \frac{3}{2}r} = \frac{4r}{4r + 3r} = \frac{4}{7}$$

12. **Soln (D)** Let x be the number of short women. The number of tall women is therefore $31 - x$. The number of men is $50 - 31 = 19$ and so the number of short men is 5. Since the number of short persons is 18, we have $5 + x = 18$. Solving, $x = 13$.

13. **Soln (D)** Let the reference person be Bob. The total in the line is

$$\begin{aligned} \text{Total} &= \text{Number behind Bob} + \text{Number in front of Bob} + \text{Bob} \\ &= (x - y) + z + 1 \\ &= z + x - y + 1. \end{aligned}$$

14. **Soln (C)** Combining parts (drink mix and water), 5 parts make 60 cups. The question is now, how many cups do 6 parts make? Let x be this amount. $\frac{5}{6} = \frac{60}{x}$. This gives $5x = 360$ and so $x = 72$.

15. **Soln (C)** Let x be the number of correct response, y the number of incorrect and z the number of questions unanswered. His score is $5x - 2y = 48$. where $x + y + z = 20$. So

$$\begin{aligned} 5x - 2y &= 48 \\ 2x + 2y + 2z &= 40 \end{aligned}$$

Adding, $7x + 2z = 88$ or $7x = 88 - 2z$. Now x is greatest when z is least. The least such z is 2 corresponding to $x = 12$

16. **Soln (B)** Let the area of the slices be $A_8, A_{10}, A_{12}, A_{14}$ respectively.

$$\begin{aligned} A_8 &= \frac{1}{3} \cdot \pi \cdot 4^2 = \frac{16}{3}\pi = 5\frac{1}{3}\pi \\ A_{10} &= \frac{1}{4} \cdot \pi \cdot 5^2 = \frac{25}{4}\pi = 6\frac{1}{4}\pi \\ A_{12} &= \frac{1}{6} \cdot \pi \cdot 6^2 = \frac{36}{6}\pi = 6\pi \\ A_{14} &= \frac{1}{8} \cdot \pi \cdot 7^2 = \frac{49}{8}\pi = 6\frac{1}{8}\pi \end{aligned}$$

Of these, the largest is $6\frac{1}{4}\pi$ corresponding to a 10-inch pizza.

17. **Soln (D)** Clearly the largest amount is received when only one person gets more than \$5 and when the other 7 persons receive as small as possible. Let m be the largest amount. We therefore want

$$m + 2 + 2 + 2 + 2 + 1 + 1 + 1 = 30$$

This corresponds to $m = 19$.

18. **Soln (A)** Suppose the original amount of money is $\$x$. Cecil receives

$$\begin{aligned} x - \left[\left(1 + \frac{2}{3}(x-1) \right) + 6 + \frac{1}{3} \left(x - \left(1 + \frac{2}{3}(x-1) \right) - 6 \right) \right] &= 40 \\ x - 1 - \frac{2}{3}x + \frac{2}{3} - 6 - \frac{1}{3}x + \frac{1}{3} + \frac{2}{9}x - \frac{2}{9} + 2 &= 40 \\ \frac{2}{9}x - \frac{38}{9} &= 40 \\ 2x - 38 &= 360 \\ 2x &= 398 \\ x &= 199 \end{aligned}$$

Byron's share is

$$\begin{aligned} 6 + \frac{1}{3} \left(x - \left(1 + \frac{2}{3}(x-1) \right) - 6 \right) &= 6 + \frac{1}{3} \left(199 - \left(1 + \frac{2}{3}(198) \right) - 6 \right) \\ &= 26 \end{aligned}$$

Alternatively, After Alyah had received his share, Byron received \$6 plus one-third of the remainder. Since Cecil gets the rest, she received two-thirds of the remainder, which is \$40. Thus, one-third of the remainder is \$20. Therefore Byron receives is $\$20 + \$6 = \$26$.

19. **Soln (B)** Let $x = 20p, y = 20q$ and $z = 20r$ so that

$$20p + 20q + 20r = 180 \text{ or } p + q + r = 9$$

and $p \leq q \leq r \in \mathbb{Z}^+$. The possible triplets are

$$(1, 1, 7), (1, 2, 6), (1, 3, 5), (1, 4, 4), (2, 2, 5), (2, 3, 4), (3, 3, 3)$$

There are 7 triplets

20. **Soln (E)** Since $3 + 8 + 6 + 9 = 26, 9 + D + 5 + 1 = 26$ and so $D = 11$. With this information,

$$A + B = 22, A + C = 14, C + E = 9 \text{ and } B + E = 17$$

The list of numbers not yet used is $\{2, 7, 10, 12\}$. We therefore have two possibilities for C and E . **Case 1:** $C = 7$ and $E = 2$. In this case B would be 15 which is not in the list. **Case 2:** $C = 2$ and $E = 7$. In this case, $B = 10$ and $A = 12$.

21. **Soln (D)** It is clear that the number of females is 99 and the number of males is 1. This male lives on campus (otherwise all the students living on campus) would be male. The 2% males living on campus is accounted for by this 1 male. The 100% living on campus would therefore be $50 \times 1 = 50$ (49 females and 1 male) There are 7 triplets

22. **Soln (D)** Because three car washers can wash 4 cars in 5 hours, 1 car washer will wash $\frac{4}{3}$ cars in 5 hours or $\frac{4}{15}$ cars in 1 hour. The 7 car washers will now work at a combined rate of $\frac{4}{15} \times 7 = \frac{28}{15}$ cars in 1 hour. If h is the number of hours to wash the 18 cars, then

$$\frac{28}{15}h = 18$$

This gives $h = \frac{18 \times 15}{28} = \frac{135}{14} = 9\frac{9}{14}$. Rounding up, we get 10.

23. **Soln (A)** Let d be the number of days in a week and the number of months be m . So $d \cdot d = d^2$ is the number of days in one month. We therefore have

$$md^2 = 1,250$$

Also,

$$m = 2d^2 \text{ of } d^2 = \frac{m}{2}$$

It follows that

$$m \cdot \frac{m}{2} = 1,250$$

This gives $m^2 = 2500$ and therefore $m = 50$.

24. **Soln (C)** We have (the sum of the proper divisors)

$$13 : 1 < 13$$

$$16 : 1 + 2 + 4 + 8 = 15 < 16$$

$$30 : 1 + 2 + 3 + 5 + 6 + 10 + 15 = 42 > 30$$

$$44 : 1 + 2 + 4 + 11 + 22 = 40 < 44$$

$$50 : 1 + 2 + 5 + 10 + 25 = 43 < 50$$

Of the list only 30 is an *interesting number*.

25. **Soln (B)** The octagon may be decomposed into 4 congruent rectangles and 8 congruent triangles. The shaded area consists of one such rectangle and 2 of the triangles. If x is the area of one of the rectangle and y is the area of one of the triangle, then the fraction is

$$\frac{x + 2y}{4x + 8y} = \frac{x + 2y}{4(x + 2y)} = \frac{1}{4}.$$