2022-2023 Senior Mathematical Olympiad

Final Round Examination (Grades 7 and 8) - Solutions

1. **Soln**: If the ratios $x : 4$ and $9 : y$ are equal, then

$$\frac{x}{4} = \frac{9}{y}$$

which gives $xy = 36$. The pairs are

$$(1,36), (2,18), (3,12), (4,9), (6,6), (9,4), (12,3), (18,2), (36,1)$$

2. **Soln**: Let the amount being shared be $\$x$. The three dollar shares are

$$\begin{aligned}
\text{Altiman} &: 100 + \frac{1}{3}(x-100) = \frac{1}{3}(x+200) \\
\text{Britannie} &: 600 + \frac{1}{3}\left(x - \frac{1}{3}(x+200) - 600\right) = \frac{2}{9}x + \frac{3400}{9} \\
\text{Cecil} &: 4000
\end{aligned}$$

Since Cecil's amount is the remainder, it represents $x - \left(\frac{1}{3}(x+200) + \frac{2}{9}x + \frac{3400}{9}\right) = \frac{4}{9}x - \frac{4000}{9}$. So

$$\frac{4}{9}x - \frac{4000}{9} = 4000$$

$4x - 4000 = 36000$. From this, $4x = 40000$ giving $x = \frac{40000}{4} = 10\ 000$.

Alternative Solution: After Britannie receives \$600, Cecil's amount represent $\frac{2}{3}$ of what is left which is \$4000. So Britannie's amount is $\$600 + \$2000 = \$2600$. Solving

$$\frac{2}{9}x + \frac{3400}{9} = 2600$$

gives $x = 10\,000$.

3. **Soln**: The area of the complete circle is $A = \pi r^2 = 25\pi$. The area of the rectangle is therefore $100 + 25\pi$ and the length of side AB is $\frac{100+25\pi}{10} = 10 + 2.5\pi$. The closest distance between the two circles is then $10 + 2.5\pi - 10 = 2.5\pi$.

4. **Soln**: If the number is divisible by 12 it must be divisible by both 3 and 4. It is divisible by 4 if the two digit number $k2$ is divisible by 4. The choices of k are $1, 3, 5, 7, 9$. If the number is divisible by 3 the sum of the digits must be divisible by 3. The sum of the digits is $16 + k$. The choices of k are $2, 5, 8$. For the number to be divisible by 12, the only possible value of k is 5 corresponding to 27552.

5. **Soln**: Because $30^2 + 40^2 = 50^2$, the triangle is right angled with hypotenuse of length 50 cm. The area of the triangle is therefore $\frac{1}{2}(30)(40) = 600$ cm^2. If h is the shortest altitude, then

$$\text{Area} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(50)\,h = 25h$$

So $600 = 25h$ and $h = \frac{600}{25} = 24$ cm.

6. **Soln**: Let $t = 2^{125} \times 3^{81} \times 5^{131}$

$$\begin{aligned} w &= 2^4\left(2^{125} \times 3^{81} \times 5^{131}\right) = 16t, \\ x &= 2^2\left(2^{125} \times 3^{81} \times 5^{131}\right) = 4t \\ y &= 2^1 \cdot 3^1\left(2^{125} \times 3^{81} \times 5^{131}\right) = 6t \\ \text{and } z &= 3^1 \cdot 5^1\left(2^{125} \times 3^{81} \times 5^{131}\right) = 15t \end{aligned}$$

This shows that $x < y < z < w$.

7. **Soln**: The area of the bigger square is n^2 cm^2 and so the number of tiles available is $T = n^2 + 92$. After the increase, the area of the bigger square is $(n+2)^2 = n^2 + 4n + 4$ and $T = n^2 + 4n + 4 - 100 = n^2 + 4n - 96$. So

$$n^2 + 92 = n^2 + 4n - 96$$

This gives $4n = 188$ or $n = \frac{188}{4} = 47$. The number of tiles available is $T = 47^2 + 92 = 2301$.