2022-2023 Senior Mathematical Olympiad

Final Round Examination (Grades 9, 10 and 11)

Provide complete solutions to all 8 questions (2 hours)

1. A sum of money is being divided among Altiman, Britannie and Cecil. First, Altiman receives $100 plus one-third of what is left. Britannie then receives $600 plus one-third of what remains. Finally Cecil receives $4000 which is the remaining amount. What is the total amount of money that is being shared?

2. Freddie has a number of square tiles, each measuring 1 cm by 1 cm. He tries to put these small square tiles together to form a larger square of side length n cm, but finds that he has 92 tiles left over. If he had increased the side length of the larger square to $(n+2)$ cm, he would have been 100 tiles short of completing the larger square. How many tiles does Freddie have?

3. What is the value or values of the digit k that makes the five-digit number

$$275k2$$

   divisible by 12?

4. A function f is such that

$$f(x) - f(x-1) = 4x - 9 \text{ and } f(5) = 18.$$

   (i) Show that $f(3) = 0$.
   (ii) If $f(x) = 2x^2 + px + q$, determine the values of p and q.

5. ABC is an equilateral triangle with sides of length 4 cm. Points P, Q and R are chosen on sides AB, BC and CA, respectively, such that $AP = BQ = CR = 1$ cm. What is the area of triangle PQR?

6. If $wxyz$ is a four-digit positive integer with $w \neq 0$, the *layer sum* of this integer equals

$$wxyz + xyz + yz + z.$$

   For example, the *layer sum* of 4089 is $4089 + 089 + 89 + 9 = 4276$.

   If the *layer sum* of $wxyz$ equals 2024, what are the possible numbers for $wxyz$?

The questions continue on the flip side of this page ......

7. The sum of **the squares** of 5 consecutive positive integers is 1815. What are the five consecutive numbers?

8. Determine the smallest positive integer N for which

$$x^4 + 2023x^2 + N$$

can be factored as $(x^2 + rx + s)(x^2 + tx + u)$ where r, s, t, u are integers and $r \neq 0$.