2022-2023 Senior Mathematical Olympiad

Round One Examination (Grades 9, 10 and 11) Solutions

1. **Soln**: **(A)** We have $(6\blacksquare 3)+4-1=5$ or $(6\blacksquare 3)=2$. Of the symbols given, $\blacksquare$ represents the $\div$ symbol.

2. **Soln**: **(A)** $(2022)(2022)^{2022}=(2022)^{1}(2022)^{2022}=(2022)^{1+2022}=(2022)^{2023}$.

3. **Soln**: **(E)** $360=3\cdot 120=3\cdot 4\cdot 30=3\cdot 4\cdot 5\cdot 6$

4. **Soln**: **(A)** $2022=2\times 1011\times 1$. This result in the largest value of $J+M+O=2+1011+1=1014$

5. **Soln**: **(B)** The number is smallest when the leading digits/cards are arranged smallest to largest from left to right. In order, they are

$$107, 31, 4, 59, 8$$

6. **Soln**: **(C)** Since angles on a straight line adds to 180°, the base angles of the triangle are 65° and 55°. The sum of the angles in a triangle is 180° and so $x=180-(65+55)=60$

7. **Soln**: **(D)** Let $QY=x$. The area below PQ is the area of the triangle plus the area of 1 square. This must be equal to 5 squares. So

$$1+\frac{1}{2}\cdot 5(1+x)=5$$

This gives $2+5(1+x)=10$ and so $1+x=\frac{8}{5}$ or $x=\frac{3}{5}$. Since $QY=x=\frac{3}{5}$, $XQ=1-x=\frac{2}{5}$ and the ratio $\dfrac{XQ}{QY}=\dfrac{2}{3}$.

8. **Soln**: **(C)** With the straight line DB, we construct two right angled triangles. The area of the quadrilateral is now

$$\frac{1}{2}(AB)(AD)+\frac{1}{2}(CD)(CB)=\frac{1}{2}(11)(3)+\frac{1}{2}(9)(7)=48$$

9. **Soln**: **(D)** Let f be the fixed monthly fee, h be the hourly rate and t, the total connect time in December. We have

$$f+ht=1248 \text{ and } f+h\cdot 2t=1754$$

From this, $2f+2ht=2496$ and $f+2ht=1754$. Subtracting, $f=2496-1754=742$.

10. **Soln**: **(C)** The perimeter of each shape is $4 \times 9 = 36$ cm. The length of one side of the triangle is therefore $\frac{36}{3} = 12$ cm. This is the length of the long side of the rectangle. Because $12 + 12 = 24$, the short sides add to give 12 cm and the length of the short side is 6 cm.

11. **Soln**: **(A)** There is only one choice for the first digit (namely 1). There are 5 choices for the second digit (namely $1, 3, 5, 7, 9$) and there are 5 choices for the third digit (namely $1, 3, 5, 7, 9$). The total is $1 \times 5 \times 5 = 25$

12. **Soln**: **(D)** Let the number under the white card be w, the number under the grey card be g and the number under the black card be b. It follows that

$$\begin{aligned} g + 2w &= 34 \\ g + w + b &= 32 \\ w + 2g &= 26 \end{aligned}$$

Adding the first and third equation, $3g + 3w = 60$. Dividing by 3, $g + w = 20$. Because $g + w = 20$, $b = 32 - 20 = 12$

13. **Soln**: **(E)** The weight of non-water in the watermelon is $\frac{2}{100} \times 20 = \frac{2}{5}$ kg. Let the present weight of the watermelon be x kg.

The $\frac{2}{5}$ kg non-water component now represents $5\% = \frac{1}{20}$ of x. The value of x is therefore $\frac{2}{5} \times 20 = 8$ kg

14. **Soln**: **(B)** Let $OX = OY = r$ and let $XY = d$. Using Pythagoras' Theorem, $d^2 = r^2 + r^2 = 2r^2$. The areas of $T, T + S$ and $S + C$ are respectively

$$\frac{1}{2} r \cdot r = \frac{1}{2} r^2, \frac{1}{4} \pi r^2 \text{ and } \frac{1}{2} \cdot \pi \frac{d^2}{4} = \frac{\pi r^2}{4}$$

It follows that

$$\frac{\text{Area of } T}{\text{Area of } C} = \frac{\frac{1}{2} r^2}{\frac{\pi r^2}{4} - \left(\frac{1}{4} \pi r^2 - \frac{1}{2} r^2\right)}$$

Multiplying through by 4 and dividing by r^2 :

$$\frac{\text{Area of } T}{\text{Area of } C} = \frac{2}{\pi - (\pi - 2)} = 1$$

15. **Soln**: **(C)** Since the area of PRS is half that of PQR it has the same area as SRQ. Because these triangles have the same area and the same altitude (from R to PQ), the bases PS and SQ are equal and hence $RS = SQ$. It now follows that $\angle QSR = 120°$ and $\angle SRQ = \angle SQR = 30°$. So $\angle PRQ = 60° + 30° = 90°$.

16. **Soln**: **(B)** Suppose Joe works at x books per hour, then Barney works at $\frac{3}{2}x$ per hour and Adam works at $\frac{5}{2}x$ per hour. For the times specified, the number of books carried is

$$3\left(\frac{5}{2}x\right)+4\left(\frac{3}{2}x\right)+5x=999$$

Multiplying by 2 :

$$15x+12x+10x=1998$$

That is, $37x=1998$ and $x=\frac{1998}{37}=54$. From this, Joe carries $5\times 54=270$, Barney carries $4\times\frac{3}{2}\times 54=324$ and Adam carries $3\times\frac{5}{2}\times 54=405$.

17. **Soln**: **(E)** We have

$$\begin{aligned}\frac{a}{b}+\frac{b}{a}-ab &= \frac{a^2+b^2-(ab)^2}{ab}=\frac{a^2+b^2-(a-b)^2}{ab}\\ &= \frac{a^2+b^2-(a^2+b^2-2ab)}{ab}=\frac{2ab}{ab}=2\end{aligned}$$

18. **Soln**: **(E)** Let k and j be the number of students in Karen and Jacqui's class. The total number of apples shared is $6k+6j$. It follows that

$$\frac{6k+6j}{k}=6+\frac{6j}{k}=10$$

From this $\dfrac{6j}{k}=4$ and $\dfrac{k}{6j}=\dfrac{1}{4}$or $\dfrac{6k}{j}=\dfrac{36}{4}=9$. The average for Jacqui's class is

$$\frac{6k+6j}{j}=6+\frac{6k}{j}=6+9=15.$$

19. **Soln**: **(D)** Since Dee is three times as old as Bea, Dee can either be 3 or 6 corresponding to Bea being 1 or 2

A	B	C	D	E	F	G
	1		3			

or

A	B	C	D	E	F	G
	2		6			

Because C=E+4, this means that C= $5, 6$ or 7. The possibilities are now

A	B	C	D	E	F	G
	1	6	3	2		

or

A	B	C	D	E	F	G
	2	5	6	1		

or

A	B	C	D	E	F	G
	2	7	6	3		

F>A and A>G means F>A>G

A	B	C	D	E	F	G
5	1	6	3	2	7	4

or

A	B	C	D	E	F	G
4	2	5	6	1	7	3

or

A	B	C	D	E	F	G
4	2	7	6	3	5	1

Since A+G>F, the only possibility is

A	B	C	D	E	F	G
5	1	6	3	2	7	4

So Aaa is 5.

20. **Soln**: **(A)**For x, y, z bags respectively, the average price per kilogram is

$$\frac{60x + 150y + 250z}{x + 3y + 8z} = 40$$

So

$$\begin{aligned} 60x + 150y + 250z &= 40x + 120y + 320z \\ 20x + 30y &= 70z \\ 2x + 3y &= 7z \end{aligned}$$

The smallest integer quantities correspond to $x = 2, y = 1, z = 1$. The smallest number is $2 + 1 + 1 = 4$

21. **Soln**: **(B)** Let the legs of the triangle be $2x$. The length of the long side is therefore $\sqrt{(2x)^2 + (2x)^2} = \sqrt{8x^2} = 2\sqrt{2}x$. The area of square P is $x^2 = 45$. The area of square R is

$$\left(\frac{2\sqrt{2}x}{3}\right)^2 = \frac{8}{9}x^2 = \frac{8}{9} \times 45 = 40$$

22. **Soln**: **(C)** The sum of the first 2 must be even, the sum of the first three must be divisible by 3, the sum of the first 4 must be divisible by 4, the sum of the first 5 must be divisible by 5. The first two numbers must be both even or both odd. On dividing the given numbers by 3, the remainders are $2, 1, 2, 1, 1$ and so the first three numbers must be $76, 82$ and 91 (91 is the third). The sum is $76 + 82 + 91 = 249$. This is one more than a multiple of 4 and so the fourth number must be 71 and the last 80.

23. **Soln**: **(B)** Because angle $ADP = 30°$ and $AD = 1, DP = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ and $AP = \frac{\sqrt{3}}{3}$. Because angle $BDC = 30°$ and $BC = 1, DB = 2$ and $DC = \sqrt{3}$. It follows that $PB = \sqrt{3} - \frac{\sqrt{3}}{3} = \frac{2\sqrt{3}}{3}$. The perimeter of ΔBDP is therefore

$$DP + PB + DB = \frac{2\sqrt{3}}{3} + \frac{2\sqrt{3}}{3} + 2 = 2 + \frac{4\sqrt{3}}{3}.$$

24. **Soln**: **(B)** Let g be the area of the grey region, w the white part of the triangle and p the area of the white part outside the triangle. We are given that

$$\frac{g}{g+w+p}=\frac{45}{100} \text{ and } \frac{w}{g+w+p}=\frac{40}{100}$$

So $\dfrac{p}{g+w+p}=\dfrac{15}{100}$. We want $\dfrac{p}{g+p}$. Taking $g+w+p$ to be 100, the ratio of $g:w:p$ is $45:40:15$ and

$$\frac{p}{g+p}=\frac{15}{45+15}=\frac{1}{4}=25\%$$

25. **Soln**: **(C)** The prime numbers between 4 and 18 are $5, 7, 11, 13, 17$. Let the chosen prime numbers be x and y so that the number obtained is

$$xy-(x+y)=xy-x-y=(x-1)(y-1)-1$$

Since the primes here are all odd, the required answer must be odd. The least value is $4\cdot 6-1=23$ and the largest is $12\cdot 16-1=191$. This eliminates all the answers but 119. Note that $(11-1)(13-1)-1=119$.