# 2023 Senior Mathematical Olympiad

## Qualifying Round Examination (Grades 7 and 8)

**NAME**_____

**GRADE**_____

**SCHOOL**_____

**STUDENT CONTACT NUMBER**_____

- EACH entry MUST be accompanied by a nominal entry fee of **J$1000**

- Be sure to staple ALL pages (including this one) together

- All entries must reach the Mathematics Department, U.W.I by
  **Wednesday 13 December 2023**

- You may deliver by **(a)** Hand **(b)** Courier **(c)** Local Mail

- The Courier address is
  **Mathematics Department, UWI**
  **Mona**
  **Kingston 7**

- The Mailing address is
  **Senior Mathematical Olympiad**
  **P.O. Box 94**
  **Mona Post Office**
  **Kingston 7**

1. The average or mean of the numbers

$$2, 9, 4, n, 2n$$

   is equal to 9. What is the value of the integer $n$?

   (A) 9     (B) 12     (C) 10     (D) 5     (E) 6

2. The pattern below is formed using a string of 5 integers by repeating itself:

$$5, -4, 3, -2, 1, 5, -4, 3, -2, 1, \ldots$$

   The sum of the first 53 integers in the list is $S$. What is the value of $S$?

   (A) 31     (B) 32     (C) 33     (D) 34     (E) 35

3. The symbols ♠, ■ and ★ appear in a $20 \times 20$ grid according to the pattern shown:



   In this order (♠s, ■s, ★s), how many ♠s, ■s, and ★s appear in the completed table?

   (A) $(132, 134, 134)$     (B) $(133, 133, 134)$     (C) $(133, 134, 133)$     (D) $(134, 132, 134)$
   (E)  $(134, 133, 133)$

4. Harry took a small pizza to the park. For his lunch, Harry ate 1/4 of the pizza. A cat came by and ate 1/3 of what Harry left behind. After that, a pigeon came and ate 1/3 of what the cat left behind. What fraction of the original pizza remained after the pigeon left?

   (A) 1/12     (B) 1/6     (C) 1/4     (D) 1/3     (E)  5/12

5. The diagram below shows a right-angled isosceles triangle, $\triangle ABC$. The points $D$ and $E$ are midpoints of $BC$ and $AB$ respectively.
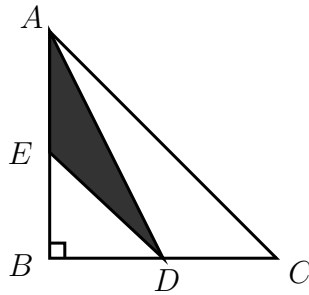


Given that $AB = BC = 24$ cm, what is the area of $\triangle AED$?

(A) 48 cm$^2$     (B) 36 cm$^2$     (C) 72 cm$^2$     (D) 9 cm$^2$     (E) 54 cm$^2$

6. The number $2023^{2023}$ is multiplying 2023 by itself 2023 times. That is,

$$2023^{2023} = \underbrace{2023 \times 2023 \times \cdots \times 2023}_{2023 \text{ times}}$$

What is the units digit of $2023^{2023}$?

(A) 1     (B) 3     (C) 5     (D) 7     (E) 9

7. At 9 a.m., Pamella finished polishing $1/2$ of her jewels. At 10 a.m., she finished polishing $7/8$ of her jewels. If Pamella polishes her jewels at a constant rate, at what time did she finish polishing all her jewels?

(A) 10:15 a.m.   (B) 11:20 a.m.   (C) 10:20 a.m.   (D) 10:30 a.m.   (E) 11:40 a.m.

8. You have \$5000 and each day you spend exactly \$60 until you can no longer do so (after day 1 you have \$4940 left). On which day, after spending, do you have exactly \$1820 left?

(A) day 41     (B) day 45     (C) day 47     (D) day 49     (E)  day 53

9. The diagram below shows a closed rectangular prism with side dimensions, 2 cm, 5 cm and 8 cm. When standing on the 2 cm by 5 cm base, the depth of the water it contains is 6 cm.
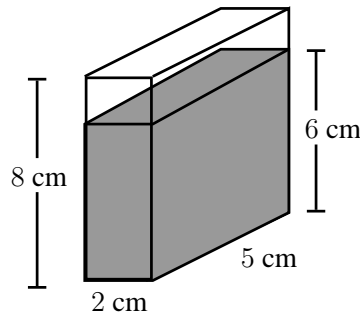


   When the prism is standing on the face with greatest area, what is the new depth of the water inside?

   (A) 0.75 cm      (B) 1 cm      (C) 1.25 cm      (D) 1.5 cm      (E) 1.75 cm

10. Tickets for a ride at a fair are sold as "combo" or "per ride". The "combo" ticket which allows for entry and unlimited rides costs \$3000. A "per ride" ticket costs \$1250 to enter and \$300 per ride. For a "combo" ticket to cost less than a "per ride" ticket, a person must go on at least how many rides?

    (A) 2      (B) 3      (C) 4      (D) 5      (E) 6

11. In a mixture, the ratio of component $A$ to component $B$ is $3 : 1$. If the amount of component $B$ is doubled and the amount of component $A$ is halved to make a new mixture, what is the ratio of component $A$ to component $B$ in this new mixture.

    (A) $3 : 2$      (B) $6 : 1$      (C) $3 : 1$      (D) $2 : 3$      (E) $3 : 4$

12. Think of a positive number. Now increase it by 60% to get a new number. This new number is now decreased by $k\%$ to get back the number you thought of. What is the value of $k$?

    (A) 57.5      (B) 40      (C) 62.5      (D) 50      (E) 37.5

13. If we multiply all the whole numbers from 1 to 50 together to get a single number, how many zeros will be at the end of this single number?

    (A) 5      (B) 10      (C) 12      (D) 20      (E) 50

14. Monty and Jonas have a total of 120 coins; Bobbie and Korrie have 153; and Monty and Bobbie have 127. In total, how many coins do Jonas and Korrie have?

    (A) 106      (B) 128      (C) 135      (D) 146      (E) 154

15. Four points, labelled $A, B, C,$ and $D$ are placed on a line in the order $A, B, C, D$. The distance between $D$ and $A$ is 24. The distance between $D$ and $B$ is 3 times the distance between $B$ and $A$. The point $C$ is halfway between $B$ and $D$. What is the distance between $C$ and $A$?

(A) 12     (B) 8     (C) 16     (D) 9     (E) 15

16. In the diagram shown, $PQR$ is a straight line segment, $\angle PQS = 125°$, $\angle QSR = x°$, and $SQ = SR$.
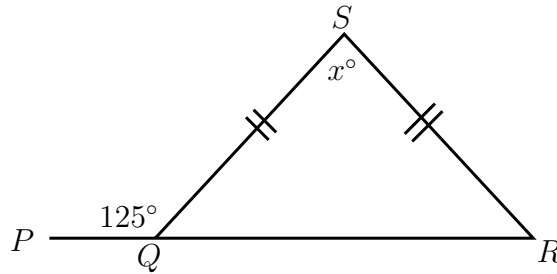


What is the value of $x$?

(A) 60     (B) 70     (C) 80     (D) 110     (E) 125

17. Paddling upstream, it took Simone 2 hours and 30 minutes to get to her destination which is 4.5 km away. Simone's return trip took her $1/3$ of the time it took her earlier. What is the average speed paddling downstream and upstream combined?

(A) 1.25 kmh$^{-1}$     (B) 3.96 kmh$^{-1}$     (C) 1.8 kmh$^{-1}$     (D) 1.95 kmh$^{-1}$     (E) 2.7 kmh$^{-1}$

18. Last February had 28 days and each night in this month Maria had a dream. Some of her dreams involved dogs, some involved cats and some involved no animals at all. You are given that 16 involved dogs, 15 involved cats and 4 involved no animals at all. How many of Maria's dreams involved BOTH dogs and cats?

(A) 3     (B) 5     (C) 7     (D) 9     (E) 11

19. On one side of a street, there are 177 trees, tree 1 to tree 177. One day while walking to school, Mark marked every other tree with a marker starting with tree 1. On his way home, he marked every third tree starting with tree 177. How many of the trees were not marked?

(A) 49     (B) 59     (C) 69     (D) 79     (E) 89

20. Gumbo gums are sold in packages of 5 gums and in packages of 8 gums. Helena can purchase exactly 18 gums by buying two 5-packs and one 8-pack, but she cannot purchase exactly 12 gums with any combination of the two packages. For how many values in the following list of possible values of $n$,

$$n = 24, 25, 26, 27, 28, 29$$

can she buy exactly $n$ gums?

(A) 5     (B) 3     (C) 2     (D) 4     (E) 6

21. Let $m$ be a positive integer. Amanda generates a sequence of terms by applying the following algorithm:

**Step 1**: Amanda writes down $m$ as the first term of the sequence

**Step 2**: If $m$ is even, she sets $n = \frac{1}{2}m$. Otherwise she sets $n = m + 1$

**Step 3**: The next term in the sequence is $m + n + 1$

**Step 4**: Amanda sets $m$ as the value written down in Step 3

**Step 5**: Amanda repeats Steps 2,3,4 until she has 5 terms

If Amanda starts with $m = 3$, what is the 5th term in her sequence?

(A) 43     (B) 44     (C) 45     (D) 46     (E) 47

22. $\overline{ab}$ and $\overline{cd}$ is one pair of two-digit numbers. If the product $\overline{ab} \times \overline{cd} = 630$, how many such pairs of $\overline{ab}$ and $\overline{cd}$ are there?

(A) 2     (B) 3     (C) 4     (D) 5     (E) 6

23. Let $(m, n)$ be a pair of positive integers that satisfy the equation

$$\frac{m}{7} + \frac{2}{n} = 1.$$

How many such pairs $(m, n)$ are there?

(A) 0     (B) 1     (C) 2     (D) 4     (E) 5

24. A square floor has a $4 \times 4$ dimension and is to be covered with 16 square tiles. The tiles are coloured red, black, green, and yellow. Each row must contain one tile of each colour and no two tiles of the same colour should share a side or share a corner. In how many ways can the floor be tiled?

(A) 256    (B) 24    (C) 120    (D) 576    (E) 64

25. A *Jayemo sequence* is a sequence of numbers in which each number after the second is the **non-negative difference (positive or zero)** between the two previous numbers. For example, if the first two numbers in a Jayemo sequence are 15 and 12, then the third number in the sequence is $15 - 12 = 3$, the fourth number is $12 - 3 = 9$, the fifth number is $9 - 3 = 6$. So the resulting sequence is $15, 12, 3, 9, 6, \ldots$. If a Jayemo sequence begins $10, 8$, what is the sum of the first 90 numbers in the sequence?

(A) 96    (B) 102    (C) 120    (D) 124    (E) 144

**Please write your name here**———————————————————————————