2024 Senior Mathematical Olympiad

Qualifying Round Examination (Grades 7 and 8)

NAME_____

GRADE_____

SCHOOL_____

STUDENT CONTACT NUMBER_____

- EACH entry MUST be accompanied by a nominal entry fee of **J$1000**
- Be sure to staple ALL pages (including this one) together
- All entries must reach the Mathematics Department, U.W.I by **Friday December 13, 2024**
- You may deliver by **(a)** Hand **(b)** Courier **(c)** Local Mail

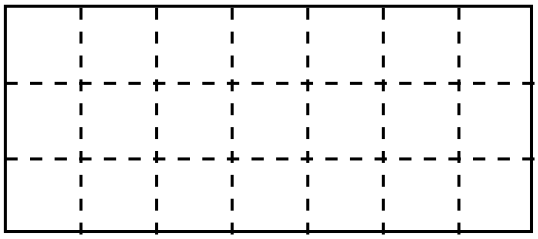
- The Courier address is
  Mathematics Department, UWI
  Mona
  Kingston 7
- The Mailing address is
  Senior Mathematical Olympiad
  P.O. Box 94
  Mona Post Office
  Kingston 7

For each question, determine the letter corresponding to the correct or best response; along with the question number, indicate this letter by circling or shading it.

1. What is the value of
$$3 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3}}}?$$
(A) $\frac{31}{10}$ (B) $\frac{49}{15}$ (C) $\frac{33}{10}$ (D) $\frac{109}{33}$ (E) $\frac{15}{4}$

2. Let x, y, z be non-zero numbers and suppose
$$(x, y, z) = \frac{xyz}{x + y + z}.$$
What is the value of $(2, 4, 6)$?
(A) 1 (B) 2 (C) 4 (D) 6 (E) 24

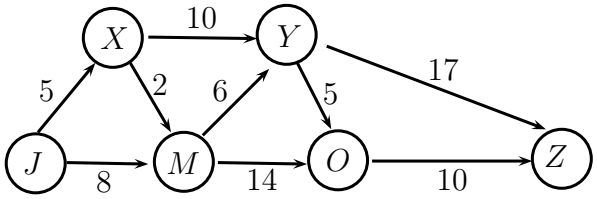
3. Pee, Que, Are, Ess and Tee ran a sprinting race. Pee beats Que, Pee beats Are, Que beats Ess and Tee finishes after Pee but before Que. Who could NOT have finished third in the race?
(A) Pee and Que (B) Pee and Are (C) Pee and Ess (D) Pee and Tee (E) Pee, Ess and Tee

4. A 3×7 flat rectangular surface is being covered with 1×1 tiles, 1×4 tiles and 2×2 tiles only.

What is the *minimum* number of 1×1 tiles that can be used?
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

5. In January of 1980, the pollution level of a certain pollutant in a pond was 338 ppm (parts per million). The pollution level increases and is expected to increase by 1.515 ppm on average yearly. To the nearest integer, in ppm, what is the expected pollution level of the pollutant in a pond in January 2030?

   (A) 399 (B) 414 (C) 420 (D) 444 (E) 259

6. Six towns J, M, O, X, Y, Z are connected via one-way streets according to the following network where the distances, in km, between connecting towns are shown.



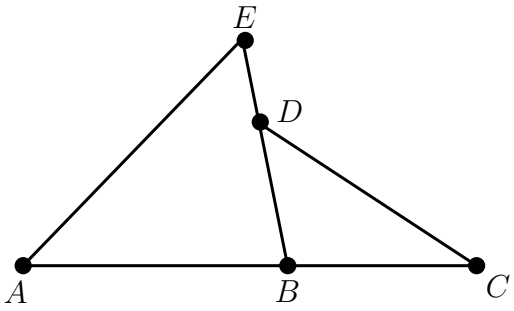
   Using the above network, what is shortest distance, in km, from J to Z?

   (A) 28 (B) 29 (C) 30 (D) 31 (E) 32

7. A gas expands by 4 cm^3 with every 3° increase in temperature. Presently, the temperature of the gas is 32° and the volume of gas present is 24 cm^3. What was the volume of the gas when the temperature was 20°?

   (A) 8 (B) 12 (C) 15 (D) 16 (E) 20

8. In the diagram below



   $\angle EAB = 60^\circ, \angle AEB = 40^\circ$ and $\angle DCB = 30^\circ$. What is the size of $\angle BDC$?

   (A) 40° (B) 50° (C) 60° (D) 70° (E) 80°

9. What fraction of points inside a circle are closer to the center of the circle than to the outer boundary of the circle?

   (A) 1/4 (B) 1/3 (C) 1/2 (D) 2/3 (E) 3/4

10. When 4 litres of water is added to a tank that is one-third full, the tank is then one-half full. What is the capacity of the tank in litres?

    (A) 8 (B) 12 (C) 20 (D) 24 (E) 48

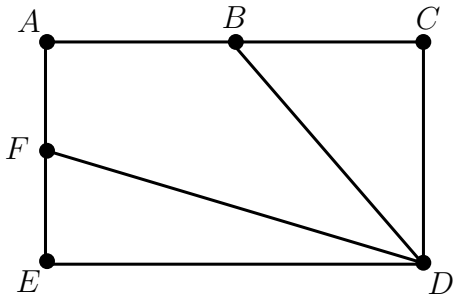
11. Petrice's driveway is covered in snow and she plans to shovel (away) all the snow. Her driveway is 4 metres wide and 10 metres long and it is covered with snow that is 3 metres deep. Based on the past, Petrice shovels at a rate of 20 cubic metres for the first hour, 19 cubic metres for the second hour, 18 cubic metres for the third hour, etc., always shovelling 1 cubic metre less per hour than she did in the previous hour. Approximately how many hours will it take Petrice to shovel away all the snow in her driveway?

    (A) 4 (B) 5 (C) 6 (D) 7 (E) 12

12. A group of frogs is called an army of frogs. An army of frogs lives in a tree and at any given time, each frog takes on one of two colours: green or yellow. A frog turns green when in the shade and turns yellow when in the sun. Initially the ratio of green to yellow frogs is $3:1$. When 3 frogs moved to the sunny side and 5 frogs moved to the shady side this ratio is $4:1$. After this movement of frogs, what is the difference between the number of green frogs and the number of yellow frogs?

    (A) 10 (B) 12 (C) 16 (D) 20 (E) 24

13. The rectangle below has length $AC = 32$ units and width $AE = 20$ units. B and F are the midpoints of AC and AE respectively.



    What is the area, in unit^2, of the quadrilateral $ABDF$?

    (A) 320 (B) 325 (C) 330 (D) 335 (E) 340

14. Let a, b, c, d be four different digits all from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. What is the smallest value of the fraction $\dfrac{a}{b} + \dfrac{c}{d}$?

    (A) 23/72 (B) 24/72 (C) 25/72 (D) 26/72 (E) 27/72

15. If the pattern continues in the sequence of letters,

$$SRJMOMJRSSRJMOMJRSSRJMOMJRSSRJMOM\cdots$$

what is the 2024th letter?

(A) S (B) R (C) J (D) M (E) O

16. The five tyres (4 road tyres and 1 spare tyre) of a car were rotated in such a way that each tyre was used the same number of kilometres during the first $30,000$ kilometers travelled by the car. For how many kilometres was each tyre used?

(A) $6,000$ (B) $7,500$ (C) $12,000$ (D) $24,000$ (E) $30,000$

17. The perimeter of square One is three times the perimeter of square Two. The area of square Two is how many times the area of square One?

(A) 2 (B) 3 (C) 4 (D) 6 (E) 9

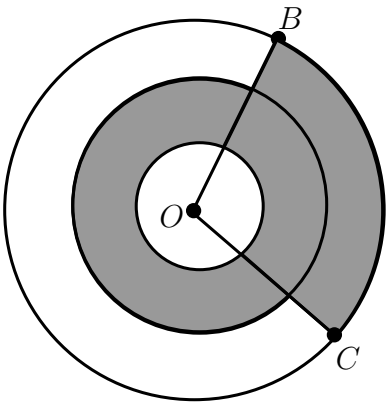
18. What percent of all the natural numbers from 1 to $10,000$ are perfect squares?

(A) 1% (B) 1.5 (C) 2% (D) 2.5% (E) 5%

19. Ramon has a large number of the digits $0, 1, 3, 4, 5, 6, 7, 8, 9$ but he has only twenty-two 2's. He is using these digits to number his very thick scrapbook starting in the order $1, 2, 3, \ldots$..How far can he number the pages of his scrapbook?

(A) 22 (B) 99 (C) 112 (D) 119 (E) 199

20. Centered at O, there are 3 concentric circles of radii $1, 2$ and 3 units. The points B and C lie on the largest circle as shown.



What is the size of $\angle BOC$ if the shaded and the unshaded regions are equal in area.

(A) 108° (B) 120° (C) 135° (D) 144° (E) 150°

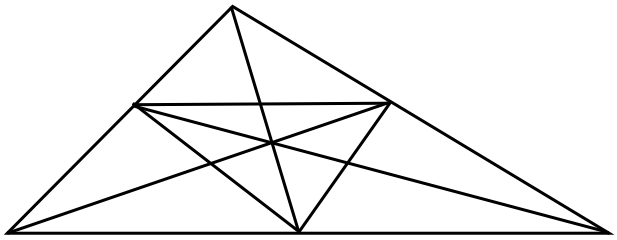
21. Harry has a calculator with a magical button. If the number x is displayed and this button is pressed, the new number displayed is $1/(1-x)$. For example, if 2 is displayed and the button is pressed once, the new display is $1/(1-2) = -1$. Suppose the calculator is currently displaying 5, what will be displayed when the magical button is pressed 100 times in a row?

(A) -0.25 (B) 0 (C) 0.8 (D) 1.25 (E) 5

22. When the length of a rectangle is increased by 20% and the width of the rectangle is increased by 50%, what is the percentage increase in the area of the original rectangle?

(A) 10 (B) 30 (C) 70 (D) 80 (E) 100

23. How many triangles are in the figure below?



(A) 24 (B) 25 (C) 47 (D) 48 (E) 72

24. On planet Weird, the occupants are called Weirdos. Weirdos have the same basic algebraic operations as us. Namely, parenthesis, addition, subtraction, multiplication and division. However, Weirdos perform parenthesis first, followed by addition and subtraction, then multiplication and then finally division, and from right to left. On planet Weird, what is the value of

$$(2-1 \div 13+16)-(5+7 \times 3-20)?$$

NOTE: On planet Weird $4 \div 2 = 1/2$ and $4-2=-2$.

(A) 175 (B) 233 (C) 224 (D) -233 (E) -175

25. What is the units digit (the rightmost digit) in the product of all the even numbers, except the 9 numbers ending in 0 (namely $10.20.30, \ldots, 90$), from 2 to 98?

(A) 2 (B) 4 (C) 6 (D) 8 (E) 0

Please write your name here_____