2024 Senior Mathematical Olympiad

Qualifying Round Examination (Grades 9, 10 and 11)

NAME———————————————————————————————

GRADE———————————————————————————————

SCHOOL———————————————————————————————

STUDENT CONTACT NUMBER————————————————————

- EACH entry MUST be accompanied by a nominal entry fee of **J\$1000**

- Be sure to staple ALL pages (including this one) together

- All entries must reach the Mathematics Department, U.W.I by
  Friday December 13, 2023

- You may deliver by **(a)** Hand **(b)** Courier **(c)** Local Mail

- The Courier address is
  Mathematics Department, UWI
  Mona
  Kingston 7

- The Mailing address is
  Senior Mathematical Olympiad
  P.O. Box 94
  Mona Post Office
  Kingston 7

For each question, determine the letter corresponding to the correct or best response; along with the question number, indicate this letter by circling or shading it.

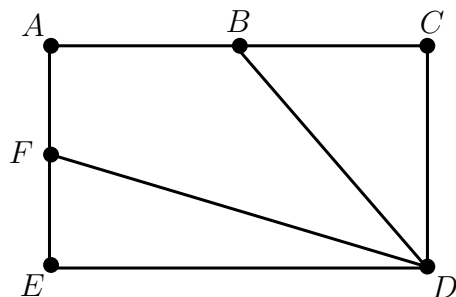1. What is the value of the product

$$\left(1+\frac{1}{1}\right) \times \left(1+\frac{1}{2}\right) \times \left(1+\frac{1}{3}\right) \times \left(1+\frac{1}{4}\right) \times \left(1+\frac{1}{5}\right) \times \left(1+\frac{1}{6}\right) ?$$

   (A) $\dfrac{7}{6}$ (B) $\dfrac{4}{3}$ (C) $\dfrac{7}{2}$ (D) 7   (E) 8

2. Six friends Al, Bebe, Coe, Dee, Eva, and Fin are arranged in that order in a circle and play a game. They start counting: Al first, then Bebe, and so forth. When the number contains a 7 as a digit (such as 47) or is a multiple of 7 that person leaves the circle and the counting continues. Who is the last person present in the circle?

   (A) Al   (B) Bebe   (C) Coe   (D) Dee   (E) Eva

3. The rectangle below has length $AC = 32$ units and width $AE = 20$ units. B and F are the midpoints of AC and AE respectively.

   What is the area, in unit^2, of the quadrilateral $ABDF$?

   (A) 320   (B) 325   (C) 330   (D) 335   (E) 340

4. Let

$$\begin{aligned} S_1 &= 1+3+5+\cdots+2021+2023 \\ \text{and } S_2 &= 2+4+6+\cdots+2022+2024 \end{aligned}$$

   What is the value of $S_2 - S_1$?

   (A) -1011 (B) -1012 (C) 1011   (D) 1012   (E) 1013

5. Let a, b, c, d be four different digits all from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. What is the smallest value of the fraction $\dfrac{a}{b} + \dfrac{c}{d}$?

(A) 23/72   (B) 24/72   (C) 25/72   (D) 26/72   (E) 27/72

6. The diagram below shows four squares with side lengths $4, 7, 9$ and 10 units with their left and bottom edges aligned.
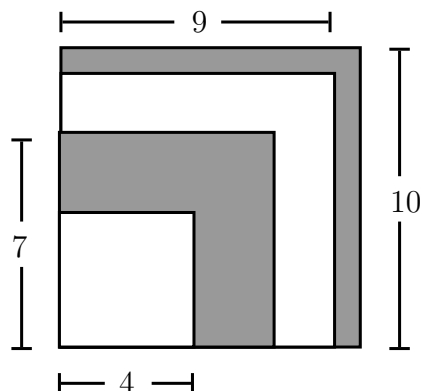

In square units, what is the (combined) area of the shaded region?

(A) 42   (B) 45   (C) 49   (D) 50   (E) 52

7. The perimeter of square One is three times the perimeter of square Two. The area of square Two is how many times the area of square One?

(A) 2   (B) 3   (C) 4   (D) 6   (E) 9

8. Starting with a 20 cm by 30 cm rectangular sheet a box is made by first removing 5 cm by 5 cm squares from each of the 4 corners. The sides are then folded to form an open box. What is the total surface area, in cm^2 of the interior of the box?

(A) 300   (B) 500   (C) 550   (D) 600   (E) 1000

9. What is the value of C in the addition problem

$$
\begin{array}{cccc}
 & A & B & C \\
 & & A & B \\
 & & & A \\
\hline
 & 3 & 0 & 0 \\
\end{array}
$$

where each letter is representing a digit?

(A) 1   (B) 3   (C) 5   (D) 7   (E) 9

10. Six towns J, M, O, X, Y, Z are connected via one-way streets according to the following network where the distances, in km, between connecting towns are shown.
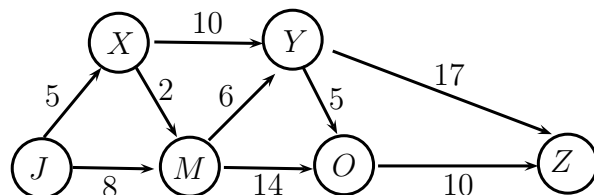
Using the above network, what is shortest distance, in km, from J to Z?

(A) 28   (B) 29   (C) 30   (D) 31   (E) 32

11. When the length of a rectangle is increased by 20% and the width of the rectangle is increased by 50%, what is the percentage increase in the area of the original rectangle?

(A) 10   (B) 30   (C) 70   (D) 80   (E) 100

12. Centered at O, there are 3 concentric circles of radii $1, 2$ and 3 units. The points B and C lie on the largest circle as shown.
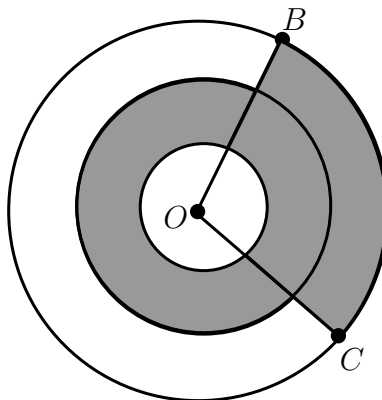
What is the size of $\angle BOC$ if the shaded and the unshaded regions are equal in area.

(A) 108°   (B) 120°   (C) 135°   (D) 144°   (E) 150°

13. Kathie owns 15 pairs of sneakers; 3/5 of the pairs are green and the rest are yellow. 2/3 of the pairs are 'bling' and the rest are 'not bling'. Of her collection, what is the least possible fraction that are green bling sneakers?

(A) 1/15   (B) 1/5   (C) 4/15   (D) 1/3   (E) 2/5

14. How many 3-digit numbers have their digits sum to 25?

    (A) 3   (B) 4   (C) 5   (D) 6   (E) 7

15. Starting on the ground, a grasshopper hops up and down a stairs, one step at a time.

    STAIRS
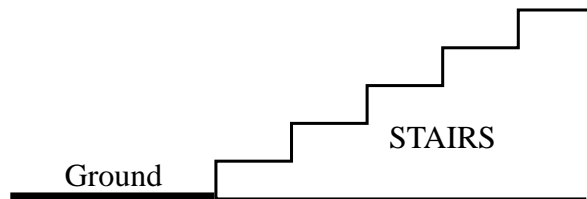
    Ground

    Making a sequence of exactly 6 hops, how many ways can the grasshopper ends back on the ground where it started?

    NOTE: One sequence is up-up-down-down-up-down (UUDDUD).

    (A) 4   (B) 5   (C) 6   (D) 8   (E) 12

16. The letters f, l, y, b, u, g represent 6 distinct (different) digits. $flyfly$ is the greatest integer satisfying the equation

    $$8 \times (flyfly) = bugbug$$

    What is the value of $fly + bug$?

    (A) 1089   (B) 1098   (C) 1107   (D) 1116   (E) 1125

17. The four digit number $2pq8$ is divisible by 9. How many different pq (in this order) are there?

    (A) 11   (B) 10   (C) 9   (D) 8   (E) 8

18. Between the fractions $1/4$ and $2/3$ are two equally spaced fractions a and b where $a < b$. What is the value of a?

    (A) 13/24   (B) 7/18   (C) 29/36   (D) 5/12   (E) 1/3

19. The manager of a company has a special fund from which she plans to give each of her employees a bonus of $5000. Unfortunately the money in the fund was short by $500 and so the manager decided that she would (instead) give each employee $4500 and leave the remaining $9500 in the fund. How much money was in the fund before the bonuses were given?

    (A) $94,500   (B) $95,000   (C) $95,500   (D) $99,000   (E) $99,500

20. There is a pattern of numbers as shown below:

$$
\begin{array}{cccccc}
 & & 1 & & & \\
 & 2 & 3 & 4 & & \\
 & 5 & 6 & 7 & 8 & 9 \\
10 & 11 & 12 & 13 & \cdots & \cdots
\end{array}
$$

If the pattern continues, what number will be the number that is directly above 280 in the array of numbers?

(A) 236   (B) 241   (C) 261   (D) 253   (E) 248

21. Consider the following sequence of numbers:

$$0, 1, 4, 5, 8, 9, 12, 13, 16, 17, \ldots$$

The 9th number in this sequence is 16 and the 10th is 17. What is the 2024th number in the sequence?

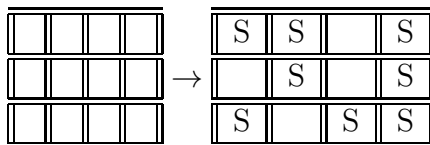(A) 1011   (B) 2023   (C) 3047   (D) 4045   (E) 5213

22. The arithmetic mean (average) of the 9 numbers

$$9, 99, 999, 9999, \ldots, 999999999$$

is m where all the digits of m are distinct (different). Which of the following is NOT a digit of m?

(A) 0   (B) 2   (C) 4   (D) 6   (E) 8

23. A small airplane has 4 rows, each with 3 seats in a row (12 seats in total), for seating passengers:

There are 8 passengers on the plane and there are 495 ways in which 8 of the 12 seats can be occupied. In how many of the 495 ways are there exactly 2 (not 3) adjacent empty seats in a row? One such seating arrangement (S) is given above.

(A) 184   (B) 216   (C) 264   (D) 300   (E) 316

24. A checkerboard consists of 1-unit squares. Ann placed a black square sheet of dimension 1.5 units by 1.5 units on the checkerboard. If n is the maximum number of checkerboard squares covered or partially covered by the black square sheet, what is the value of n?

(A) 4   (B) 6   (C) 8   (D) 10   (E) 12

25. Consider the product

$$\underbrace{9999\cdots99}_{94 \text{ nines}} \times \underbrace{4444\cdots44}_{94 \text{ fours}}$$

where a string of 94 nines is multiplied by a string of 94 fours.

What is the sum of all the digits in the resulting answer obtained from this product?

(A) 846   (B) 855   (C) 945   (D) 954   (E) 1072

Please write your name here‗‗