The University of the West Indies, Mona

presents

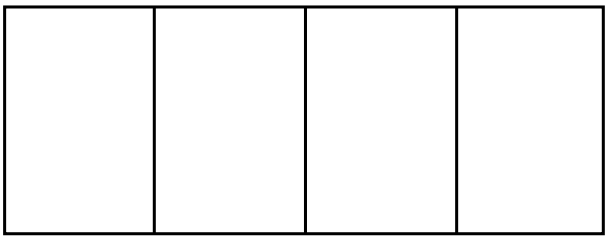
2024-2025 Senior Mathematical Olympiad

Final Round Examination (Grades 9, 10 and 11) - 11:00am

Provide complete solutions to all 10 questions (2.5 hours)

1. You are given that
$$7200 = 2^a \times\ 3^b \times\ 4^c \times\ 5^d,$$
where a, b, c and d are all positive integers. Given that $a + b + c + d = 7$, determine the value of c?

2. A rectangle, called L for large, is divided into 4 identical smaller rectangles as shown below.



The perimeter of L is 18 metres more than the perimeter of each of the smaller rectangles. The area of L is 18 m^2 more than the area of each of the smaller rectangles. Determine is the perimeter in metres of L?

3. Let m be a positive integer. The expression $m!$, which is called m factorial, is the product of the integers from 1 to m, inclusive. For example,
$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120 \text{ and } 73! = 73(72)(71)\cdots(3)(2)(1).$$
Determine the value n if
$$\frac{99!}{101! - 99!} = \frac{1}{n}.$$

4. Matthew planned a bike ride to cover a distance of 210 km, riding at a given average speed. However, Matthew rode 5 kmh^{-1} faster than he planned and finished his ride 1 hour earlier than planned. What was his average speed for the ride?

5. **NOTE**: The four-digit number $pqrs$ is divisible by 3 if 3 is a factor of $p + q + r + s$. It is divisible by 11 if $p - q + r - s$ is divisible by 11.

   Determine the five-digit number of the form $aa4ab$ that is (exactly) divisible by 165.

6. In 2024, the ratio of the number of males enrolled at the JMO University to the number of females enrolled at the JMO University was $19 : 25$. This year, the total number of students enrolled at the JMO University has not changed, but the number of females enrolled has increased by 10%. What is the current ratio of the number of males enrolled to the number of females enrolled at the JMO University?

7. The number 75 be expressed as the sum of m consecutive positive integers, where $m \geq 2$. Determine the possible values of m?

8. An auditorium has 2025 people consisting of women and men (only). It is known that exactly 1 man is taller than exactly 1 woman, exactly 1 man is taller than exactly 3 women, exactly 1 man is taller than exactly 5 women, exactly 1 man is taller than exactly 7 women and so on (with each successive man being taller than exactly two more women than the previous man). The final man is taller than all the women. How many men are in the auditorium?

9. Mrs Banks is paying a bill using internet banking. The bill is for the four-digit whole number of dollars, $\$ab00$. Mrs Banks accidentally inserts an extra digit after the b but before the adjacent 0. Later, she is rather annoyed to discover that she has overpaid the bill by \$64700. How many dollars should she have paid?

10. Anansi the spider is currently at the point O and travels to the point P in the following manner: Anansi walks 1 metre east from O, then 2 metres north, then 3 metres west, then 4 metres south, then 5 metres east, then 6 metres north, and so on, until she finally walks 41 metres east to finish at P. What is the length of OP?