The University of the West Indies, Mona

presents

2024-2025 Senior Mathematical Olympiad

**Round Two Examination (Grades 7 and 8) - 11:00am**

SECTION A

For each question, determine the letter corresponding to the correct or best response; along with the question number, indicate this letter by shading it on the answer sheet

1. What is the value of
$$\frac{(2025)^2 + 2025(2030)}{2025 + 2030}?$$

   (A) 2025   (B) 2030   (C) 4050   (D) 4055   (E) 4060

2. In triangle PQR, the lengths of sides PQ and PR are the same. The point S lies on QR so that $QS = PS$ and $\angle RPS = 75°$.
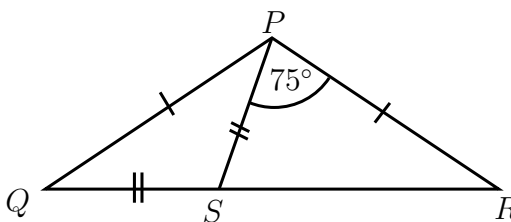
   What is the size of $\angle QRP$?

   (A) 15°   (B) 20°   (C) 25°   (D) 30°   (E) 35°

3. Today, Wendy-Ray realised the following are true:

   (i) In two years' time her brother Ted will be twice as old as he was two years ago

   (ii) In three years' time her sister Tina will be three times as old as she was three years ago.

   Which of the following statements is true?

   (A) Ted is two years older than Tina   (B) Ted is one year older than Tina
   (C) Ted is the same age as Tina   (D) Ted is one year younger than Tina
   (E) Ted is two years younger than Tina

4. Six standard, fair dice are rolled once. The total of the scores shown is 32. What is the smallest possible score that could have appeared on any of the dice?

   (A) 1   (B) 2   (C) 3   (D) 4   (E) 5

5. The diagrams below show an isosceles triangle, a right-angled triangle and a square of side length $\left(\frac{a}{4}\right)^2$ cm $= b^4$ cm.
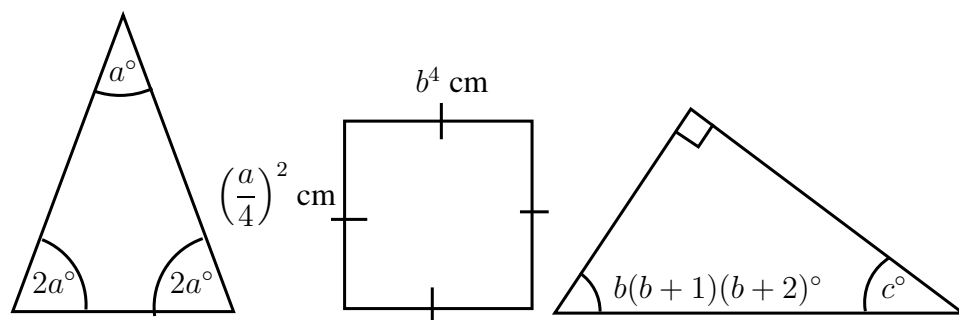
   Using the information in the diagrams above, what is the value of c?

   (A) 15   (B) 20   (C) 25   (D) 30   (E) 35

6. The product $8000 \times K$ is a square number, where K is a positive integer. What is the smallest possible value of K?

   (A) 1   (B) 2   (C) 3   (D) 4   (E) 5

7. Like most digital clocks, Maria's 24-hour digital clock displays hours and minutes only. For how many different times, during a 24-hour period, does the display contain at least one occurrence of the digit 5?

   (A) 252   (B) 372   (C) 450   (D) 512   (E) 550

8. Danielle wrote down the integers from 1 to N. She used the digit 1 fifteen times. She used the digit 2 fourteen times. What is the value of N?

   (A) 41   (B) 32   (C) 42   (D) 31   (E) 51

9. It takes one minute for a train travelling at constant speed to pass completely through a tunnel that is 120 metres long. The same train, travelling at the same constant speed, takes 20 seconds from the instant its front enters the tunnel to when the back of the train enters the tunnel. In metres, how long is the train?

   (A) 50   (B) 60   (C) 70   (D) 80   (E) 90

10. Ten people decide to hire a van together, sharing the cost equally. If there had been five more persons, the cost to each person would have been $1000 less. What is the cost of hiring the car?

   (A) \$15 000   (B) \$30 000   (C) \$50 000   (D) \$60 000   (E) \$75 000

SECTION B

For each question, provide a complete solution by showing all your workings.

1. Express
$$\frac{4^{2025} \times 3^{2026}}{6^{2026} \times 2^{2025}}$$
   in its simplest form.

2. In the following word-sum, each letter stands for one of the digits 0 to 9, and stands for the same digit each time it appears. Different letters stand for different digits and no number starts with 0.

$$
\begin{array}{r}
J \quad M \quad O \\
J \quad M \quad O \\
+ \quad J \quad M \quad O \\
\hline
S \quad M \quad O
\end{array}
$$

   Find all the possible values of the number "JMO".

3. The diagram below shows a right-angled triangle ACD with a point B on the side AC. The sides of triangle ABD have lengths $3, 7$ and 8 (units).
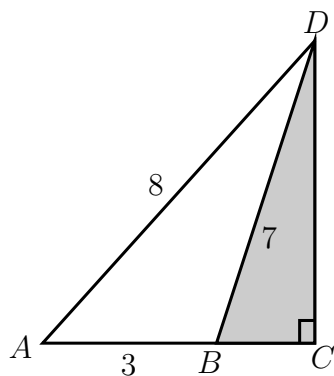
What is the area of triangle BCD?

4. In the diagram, $ABCD$ is a rectangle with $CD = 3$ units and $DA = 5$ units. A circle of radius 1 unit is centred at A, a circle of radius 2 units is centred at B and a circle of radius 3 units is centred at C.
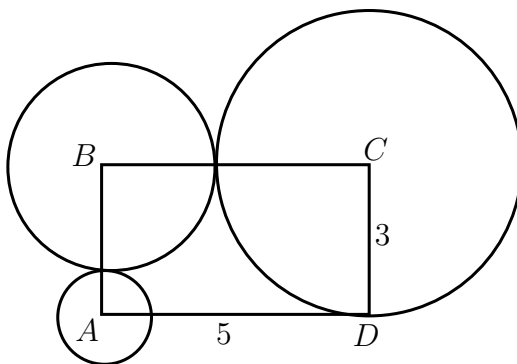
Determine the area of the region that is inside the rectangle but outside of the three circles.

5. Determine all positive integers n, such that

$$\frac{n}{30 - n}$$

is a positive integer.